Introduction to Structural Analysis

1.1 Structural Analysis Defined

A structure, as it relates to civil engineering, is a system of interconnected members used to support external loads. Structural analysis is the prediction of the response of structures to specified arbitrary external loads. During the preliminary structural design stage, a structure's potential external load is estimated, and the size of the structure's interconnected members are determined based on the estimated loads. Structural analysis establishes the relationship between a structural member's expected external load and the structure's corresponding developed internal stresses and displacements that occur within the member when in service. This is necessary to ensure that the structural members satisfy the safety and the serviceability requirements of the local building code and specifications of the area where the structure is located.

1.2 Types of Structures and Structural Members

There are several types of civil engineering structures, including buildings, bridges, towers, arches, and cables. Members or components that make up a structure can have different forms or shapes depending on their functional requirements. Structural members can be classified as beams, columns and tension structures, frames, and trusses. The features of these forms will be briefly discussed in this section.

1.2.1 Beams

Beams are structural members whose longitudinal dimensions are appreciably greater than their lateral dimensions. For example, the length of the beam, as shown in Figure 1.1, is significantly greater than its breadth and depth. The cross section of a beam can be rectangular, circular, or triangular, or it can be of what are referred to

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as standard sections, such as channels, tees, angles, and I-sections. Beams are always loaded in the longitudinal direction.

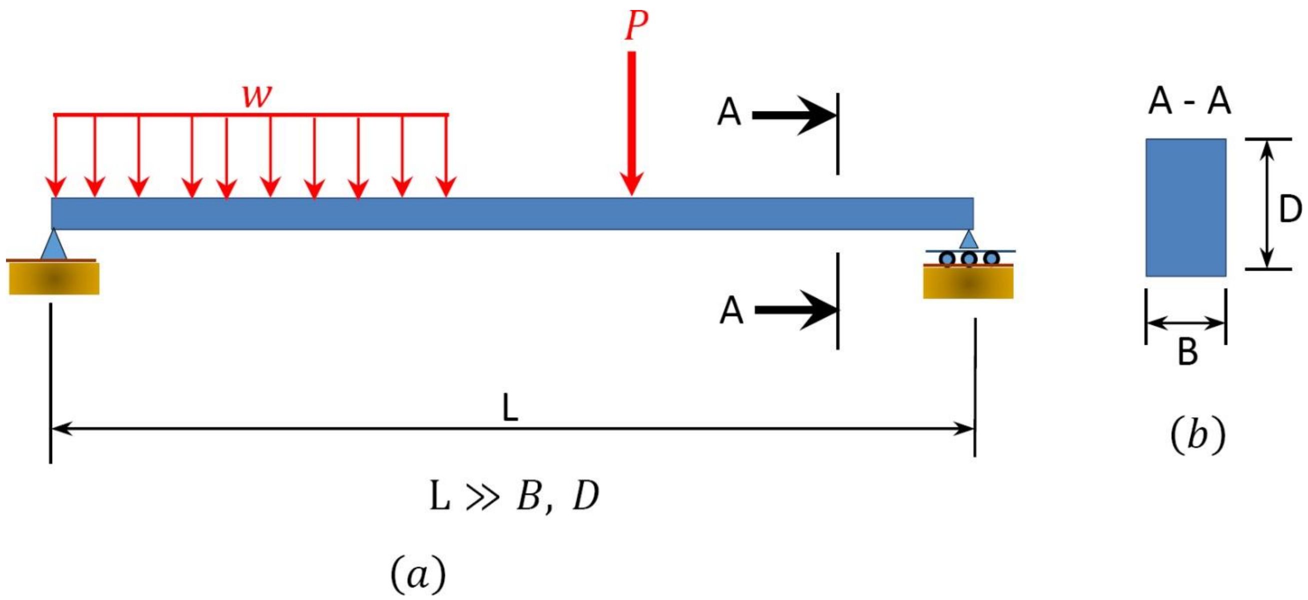

Fig. 1.1. Beam.

1.2.2 Columns and Tension Structures

Columns are vertical structural members that are subjected to axial compression, as shown in figure 1.2a. They are also referred to as struts or stanchions. Columns can be circular, square, or rectangular in their cross sections, and they can also be of standard sections. In some engineering applications, where a single-member strength may not be adequate to sustain a given load, built-up columns are used. A built-up column is composed of two or more standard sections, as shown in Figure 1.2b. Tension structures are similar to columns, with the exception that they are subjected to axial tension.

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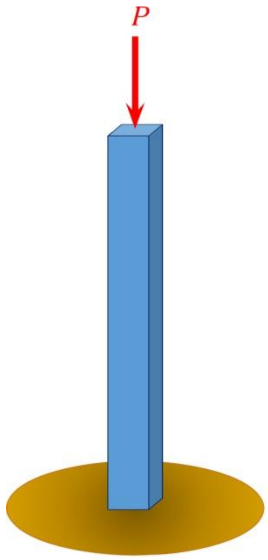
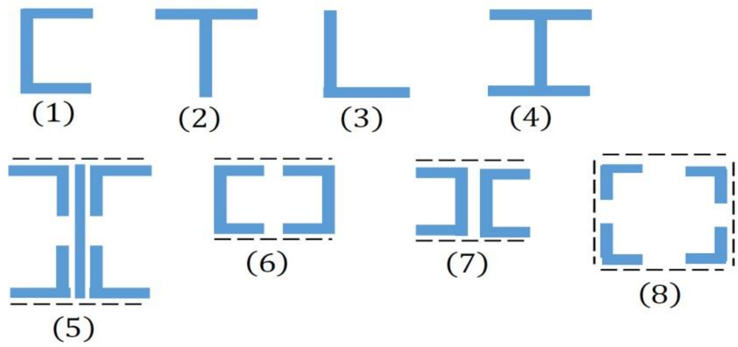


(a)



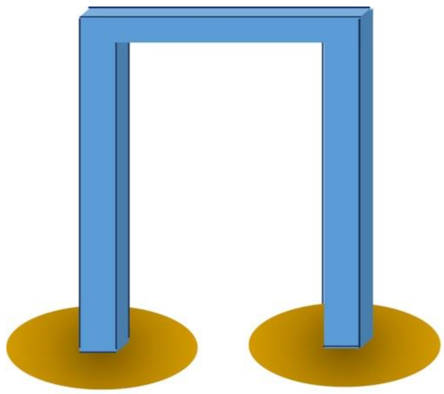
Standard sections: (1) channel, (2) tee, (3) angle, and (4) wide flange beam; Built-up sections: (5) four angles and a plate, (6) two channels (front-to-front), (7) two channels (back-to-back), and (8) four angles (front-to-front).

(b)

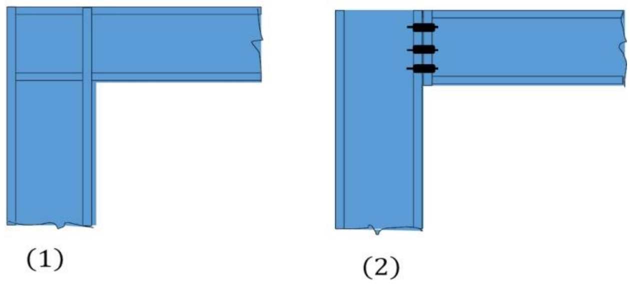
Fig. 1.2. Columns.

1.2.3 Frames

Frames are structures composed of vertical and horizontal members, as shown in Figure 1.3a. The vertical members are called columns, and the horizontal members are called beams. Frames are classified as sway or non-sway. A sway frame allows a lateral or sideward movement, while a non-sway frame does not allow movement in the horizontal direction. The lateral movement of the sway frames are accounted for in their analysis. Frames can also be classified as rigid or flexible. The joints of a rigid frame are fixed, whereas those of a flexible frame are moveable, as shown in Figure 1.3b.



(a)



Types of Frame Joints (1)Rigid or fixed joint
(2) Flexible or pinned joint

(b)

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Fig. 1.3. Frame.

1.2.4 Trusses

Trusses are structural frameworks composed of straight members connected at the joints, as shown in Figure 1.4. In the analysis of trusses, loads are applied at the joints, and members are assumed to be connected at the joints using frictionless pins.

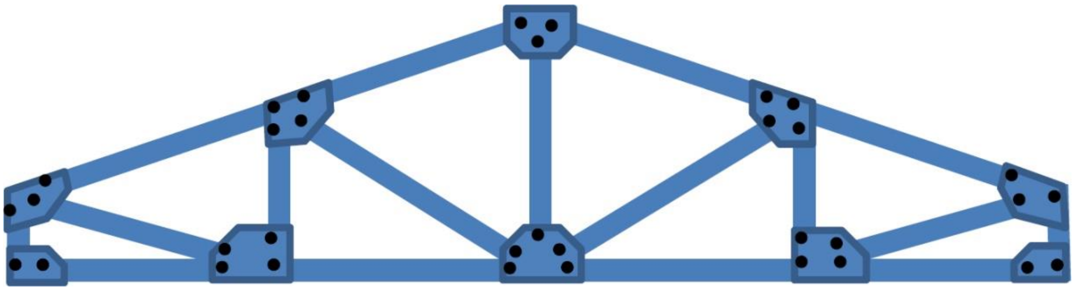

Fig. 1.4. Truss.

1.3 Fundamental Concepts and Principles of Structural Analysis

1.3.1 Equilibrium Conditions

Civil engineering structures are designed to be at rest when acted upon by external forces. A structure at rest must satisfy the equilibrium conditions, which require that the resultant force and the resultant moment acting on a structure be equal to zero. The equilibrium conditions of a structure can be expressed mathematically as follows:

$$\sum F = 0, \text{ and } \sum M = 0$$

1.3.2 Compatibility of Displacement

The compatibility of displacement concept implies that when a structure deforms, members of the structure that are connected at a point remain connected at that point without void or hole. In other words, two parts of a structure are said to be compatible in displacements if the parts remain fitted together when the structure

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deforms due to the applied load. Compatibility of displacement is a powerful concept used in the analysis of indeterminate structures with unknown redundant forces in excess of the three equations of equilibrium. For an illustration of the concept, consider the propped cantilever beam shown in Figure 1.5a. There are four unknown reactions in the beam: the reactive moment, a vertical and horizontal reaction at the fixed end, and another vertical reaction at the prop at point *B*. To determine the unknown reactions in the beam, one more equation must be added to the three equations of equilibrium. The additional equation can be obtained as follows, considering the compatibility of the structure:

$$\Delta_{BP} + \Delta_{BR} = 0$$

In this equation, Δ_{BP} is the displacement at point *B* of the structure due to the applied load *P* (Figure 1.5b), and Δ_{BR} is the displacement at point *B* due to the reaction at the prop *R* (Figure 1.5c). Students should always remember that the first subscript of the displacement indicates the location where the displacement occurs, while the second subscript indicates the load causing the displacement.

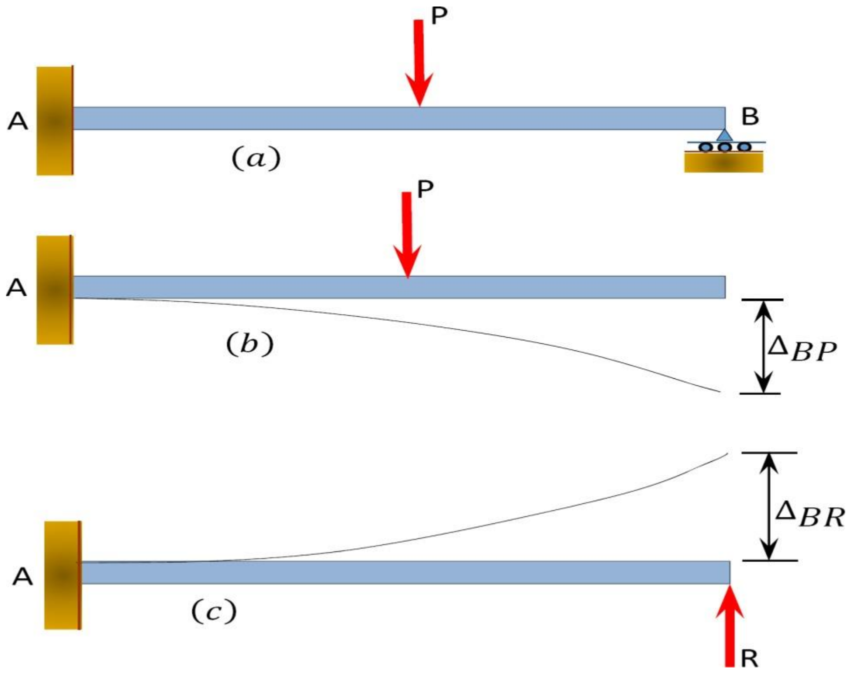

Fig. 1.5. Propped cantilever beam.

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1.3.3 Principle of Superposition

The principle of superposition is another very important principle used in structural analysis. The principle states that the load effects caused by two or more loadings in a linearly elastic structure are equal to the sum of the load effects caused by the individual loading. For an illustration, consider the cantilever beam carrying two concentrated loads P_1, and P_2, in Figure 1.6a. Figures 1.6b and 1.6c are the responses of the structure in terms of the displacement at the free end of the beam when acted upon by the individual loads. By the principle of superposition, the displacement at the free end of the beam is the algebraic sum of the displacements caused by the individual loads. This can is written as follows:

$$\Delta_B = \Delta_{BP_1} + \Delta_{BP_2}$$

In this equation, Δ_B is the displacement at B; Δ_{BP1} and Δ_{BP2} are the displacements at B caused by the loads P_1 and P_2, respectively.

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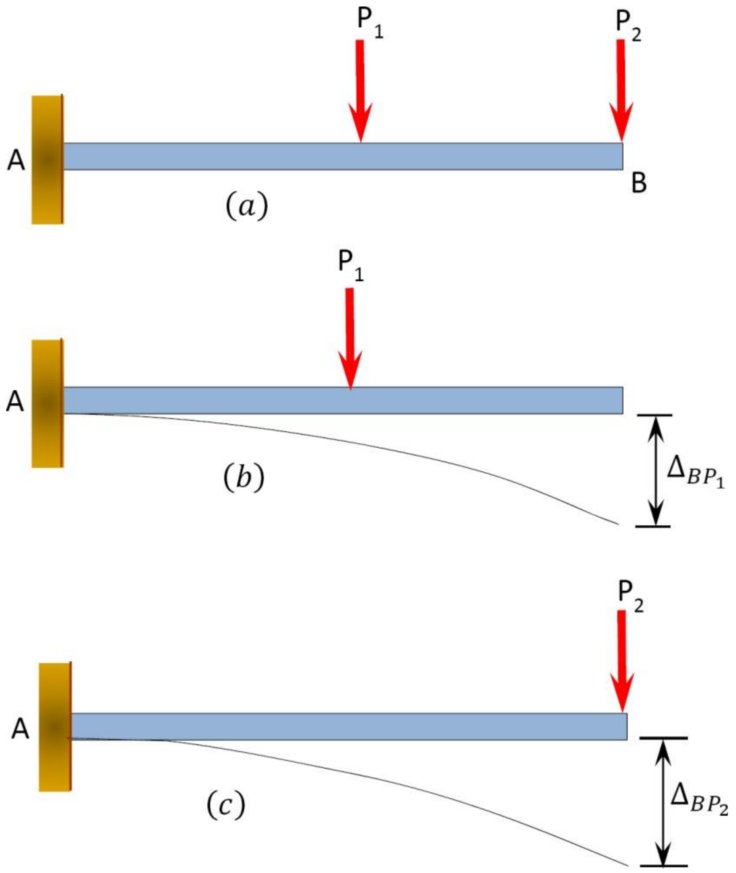

Fig. 1.6. Application of the principle of superposition.

1.3.4 Work-Energy Principle

The work-energy principle is a very powerful tool in structural analysis. Work is defined as the product of the force and the distance traveled by the force, while energy is defined as the ability to do work. Work can be transformed into various energy, including kinetic energy, potential energy, and strain energy. In the case of a structural system, based on the law of conservation of energy, work done W is equal to the strain energy U stored when deforming the system. This is expressed mathematically as follows:

$$W = U$$

Consider a case where a force F is gradually applied to a deformable structural system. By plotting the applied force against the deformation Δ of the structure, the load-deformation plot shown

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in Figure 1.7a is created. In the case of linearly elastic structure, the load-deformation diagram will be as shown in Figure 1.7b. The incremental work done dW by the force when deforming the structure over an incremental displacement $d\Delta$ is expressed as follows:

$$dW = Fd\Delta$$

The total work done is represented as follows:

$$W = \int_0^{\Delta} dW = \int_0^{\Delta} Fd\Delta$$

Thus, the strain energy is written as follows:

$$U = \int_0^{\Delta} Fd\Delta$$

The strain energy in the case of linearly elastic deformation can be obtained by computing the area under the load-deformation diagram in Figure 1.7b. This is expressed as follows:

$$U = \frac{1}{2}F\Delta$$

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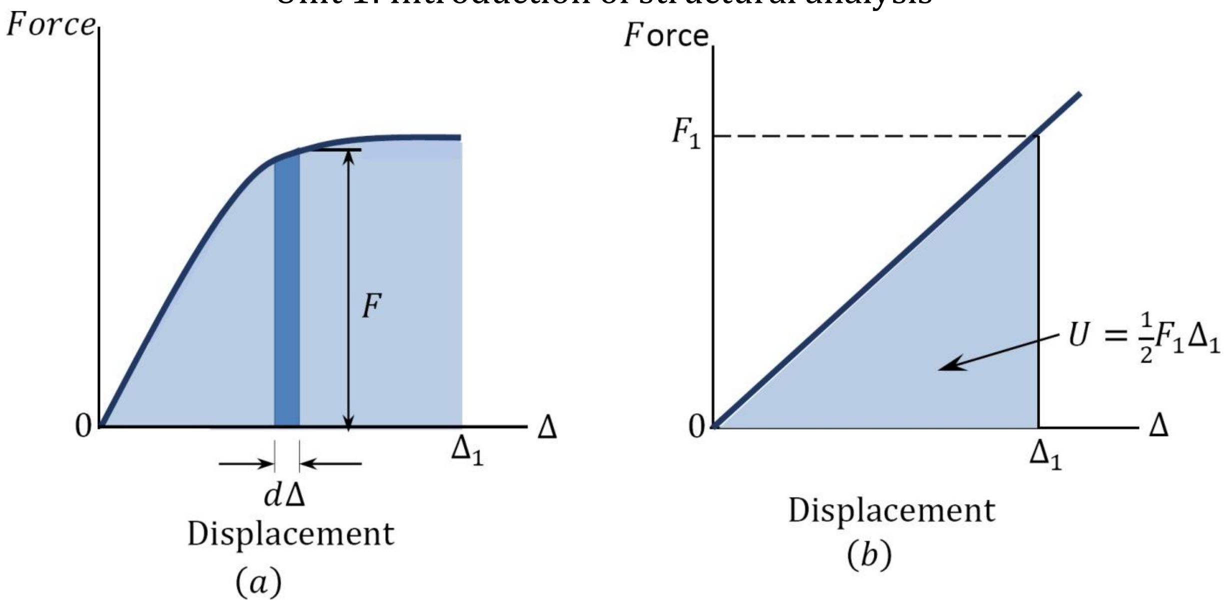

Fig. 1.7. Load-deformation diagram.

1.3.5 Virtual Work Principle

The virtual work principle is another powerful and useful analytical tool in structural analysis. It was developed in 1717 by Johann Bernoulli. Virtual work is defined as the work done by a virtual or imaginary force acting on a deformable body through a real distance, or the work done by a real force acting on a rigid body through a virtual or fictitious displacement. To formulate this principle in the case of virtual displacements through a rigid body, consider a propped cantilever beam subjected to a concentrated load P at a distance x from the fixed end, as shown in Figure 1.8a. Suppose the beam undergoes an elementary virtual displacement δu at the propped end, as shown in Figure 1.8b. The total virtual work performed is expressed as follows:

$$\delta W = R_B \delta_u - P\frac{X}{L}\delta u$$

Since the beam is in equilibrium, $\delta W = 0$ (by the definition of the principle of virtual work of a body).

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The principle of virtual work of a rigid body states that if a rigid body is in equilibrium, the total virtual work performed by all the external forces acting on the body is zero for any virtual displacement.

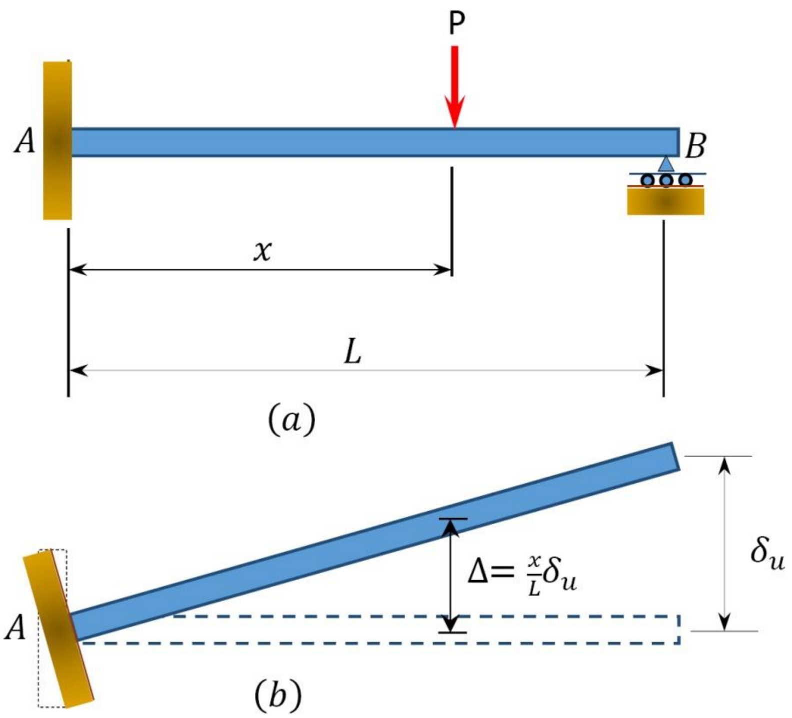

Fig. 1.8. Propped cantilever beam.

1.3.6 Structural Idealization

Structural idealization is a process in which an actual structure and the loads acting on it are replaced by simpler models for the purpose of analysis. Civil engineering structures and their loads are most often complex and thus require rigorous analysis. To make analysis less cumbersome, structures are represented in simplified forms. The choice of an appropriate simplified model is a very important aspect of the analysis process, since the predictive response of such idealization must be the same as that of the actual structure. Figure 1.9a shows a simply supported wide-flange beam structure and its load. The plan of the same beam is shown in Figure 1.9b, and the idealization of the beam is shown in Figure 1.9c. In the idealized form, the beam is represented as a line along the beam’s neutral axis, and the load acting on the beam is shown as a point or concentrated load because the load occupies an area that is significantly less than

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the total area of the structure's surface in the plane of its application. Figures 1.10a and 1.10b depict a frame and its idealization, respectively. In the idealized form, the two columns and the beam of the frame are represented by lines passing through their respective neutral axes. Figures 1.11a and 1.11b show a truss and its idealization. Members of the truss are represented by lines passing through their respective neutral axes, and the connection of members at the joints are assumed to be by frictionless pins.

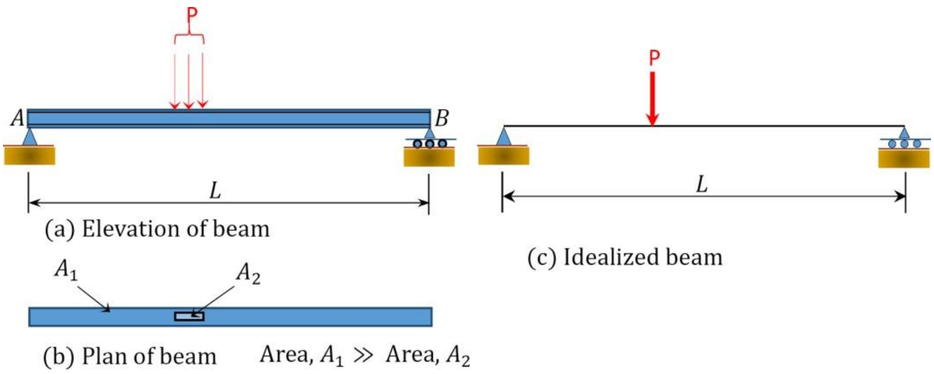

Fig. 1.9. Wide – flange beam idealization.

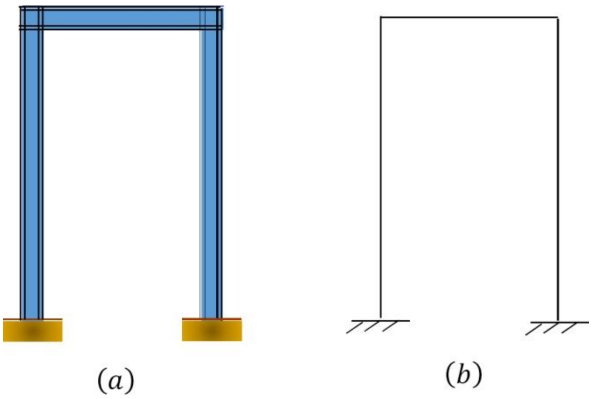

Fig. 1.10. Frame idealization.

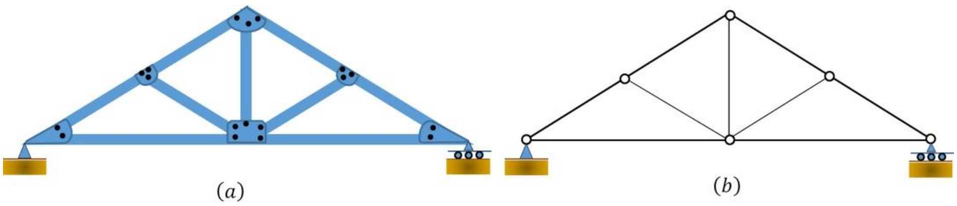

Fig. 1.11. Truss idealization.

1.3.7 Method of Sections

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The method of sections is useful when determining the internal forces in structural members that are in equilibrium. The method involves passing an imaginary section through the structural member so that it divides the structure into two parts. Member forces are determined by considering the equilibrium of either part. For a beam in equilibrium that is subjected to transverse loading, as shown in Figure 1.12, the internal forces include an axial or normal force, *N*, shear force, *V*, and bending moments, *M*.

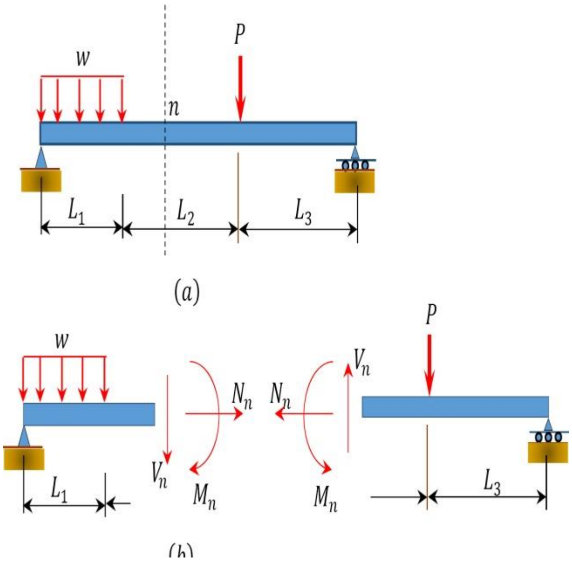

Fig. 1.12. Beam in equilibrium subjected to transverse loading.

1.3.8 Free-Body Diagram

A free-body diagram is a diagram showing all the forces and moments acting on the whole or a portion of a structure. A free-body diagram must also be in equilibrium with the actual structure. The free-body diagram of the entire beam shown in Figure 1.13a is depicted in Figure 1.13b. If the free-body diagram of a segment of the beam is desired, the segment will be isolated from the entire beam using the method of sections. Then, all the external forces on the segment and the internal forces from the adjoining part of the structure will be applied to the isolated part.

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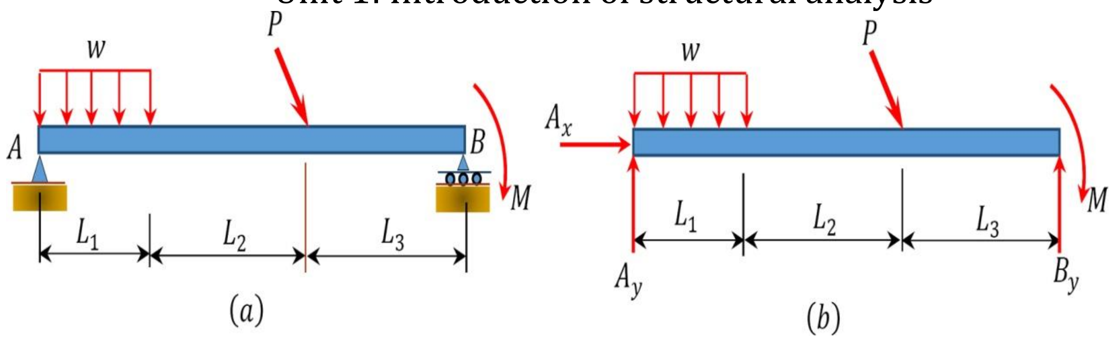

Fig. 1.13 Freebody diagram of a beam.

1.4 Units of Measurement

The two most commonly used systems in science and technology are the International System of Units (SI Units) and the United States Customary System (USCS).

1.4.1 International System of Units

In the SI units, the arbitrarily defined base units include meter (m) for length, kilogram (kg) for mass, and second (s) for time The unit of force, newton (N), is derived from Newton's second law. One newton is the force required to give a kilogram of mass an acceleration of 1 m/s^2. The magnitude, in newton, of the weight of a body of mass *m* is written as follows:

$$W\ (N) = m\ (kg) \times g\ (m/s^2)$$

where

$g = 9.81\ m/s^2$

1.4.2 United States Customary System

In the United States Customary System, the base units include foot (ft) for length, second (s) for time, and pound (lb) for force. The slug for mass is a derived unit. One slug is the mass accelerated at 1 ft/s^2 by a force of 1 lb. The mass of a body, in slug, is determined as follows:

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$$m\ (\text{slugs}) = \frac{W\ (lb)}{g\ (\frac{ft}{s^2})}\ ,\ \text{where } g = 32.2\ \text{ft/s}^2$$

The two systems of units are summarized in Table 1.1 below.

Table 1.1. Comparison of unit measurement systems.

Quantity	Length	Time	Mass	Force
Dimensional Symbol	L	T	M	F
U.S. Customary Units	foot (ft)	second (s)	Slug	pound (lb)
SI Units	meter (m)	second (s)	kilogram (kg)	Newton (N)

Table 1.2. Unit conversion.

Quantity	U.S. Customary Unit	Equal	SI Unit
Acceleration	ft/s^2		$0.3048\ m/s^2$
Area	in^2		$645.2\ mm^2$
Density	lb/ft^3		$16.02\ kg/m^3$
Energy, Work	in.lb		0.113 N.m (Joule, J)
Force	lb		4.448 N
	kip		4.448 kN
Impulse	lb.s		4.448 N.s
Length	in		25.4 mm
	ft		0.3048 m
Mass	Slug		14.59 kg
Moment of a couple	lb.in		0.113 N.m
	k.ft		1356 N.m
Moment of inertia of area	in^4		$0.4162 \times 10^{-6}\ m^4$
	ft^4		$8.6303\ x\ 10^{-3}\ m^4$
Moment of inertia of mass	$lb.ft.s^2$		$1.356\ kg.m^2$
Momentum	lb.s		4.448 kg.m/s
Power	ft.lb/s		1.356 W
Pressure	psi		6.895 kPa
	ksi		6.895 MPa
Velocity	ft/s		0.3048 m/s
Volume of an object	ft^3		$0.02832\ m^3$
Volume of a liquid	gal		3.785 L

1.4.3 SI Prefixes

Prefixes are used in the International System of Units when numerical quantities are quite large or small. Some of these prefixes are presented in Table 1.3.

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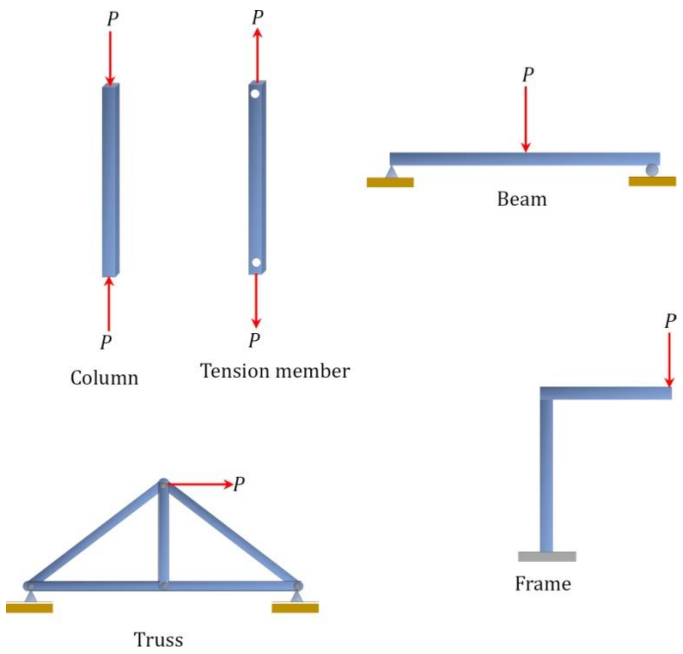
Table 1.3. SI prefixes.

Multiplication Factor	Exponential Form	Prefix	Symbol
1 000 000 000 000	10^{12}	Tera	T
1 000 000 000	10^{9}	Giga	G
1 000 000	10^{6}	Mega	M
1 000	10^{3}	Kilo	K
0.001	10^{-3}	Milli	M
0.000 001	10^{-6}	Micro	M
0.000 000 001	10^{-9}	Nano	N

Chapter Summary

Introduction to structural analysis: Structural analysis is defined as the prediction of structures' behavior when subjected to specified arbitrary external loads.

Types of structures: Structural members can be classified as beams, columns and tension structures, frames, and trusses.



Fundamental concepts of structural analysis: The fundamental concept and principles of structural analysis discussed in the chapter include equilibrium conditions, compatibility of displacement, principle of superposition, work-energy principle, virtual work principle, structural idealization, method of sections, and free-body diagram.

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Equilibrium Structures, Support Reactions, Determinacy and Stability of Beams and Frames

Equilibrium of Structures

Engineering structures must remain in equilibrium both externally and internally when subjected to a system of forces. The equilibrium requirements for structures in two and three dimensions are stated below.

3.1.1 Equilibrium in Two Dimensions

For a structure subjected to a system of forces and couples which are lying in the *xy* plane to remain at rest, it must satisfy the following three equilibrium conditions:

$$\sum F_x = 0; \sum F_y = 0; \sum M_z = 0 \qquad (3.1)$$

The above three conditions are commonly referred to as the equations of equilibrium for planar structures. $\sum F_x$ and $\sum F_y$ are the summation of the *x* and *y* components of all the forces acting on the structure, and $\sum M_z$ is the summation of the couple moments and the moments of all the forces about an axis *z*, perpendicular to the plane *xy* of the action of the forces.

3.1.2 Equilibrium in Three Dimensions

A structure in three dimensions, that is, in a space, must satisfy the following six requirements to remain in equilibrium when acted upon by external forces:

$$\sum F_x = 0; \sum F_y = 0; \sum F_z = 0$$

$$\sum M_x = 0; \sum M_y = 0; \sum M_z = 0 \qquad (3.2)$$

3.2 Types of Supports and Their Characteristics

The type of support provided for a structure is important in ensuring its stability. Supports connect the member to the ground or to some other parts of the structure. It is assumed that the student is already familiar with several types of supports for rigid bodies, as this was introduced in the statics course. However, the characteristics of some of the supports are described below and shown in Table 3.1.

3.2.1 Pin or Hinge Support

A pin support allows rotation about any axis but prevents movement in the horizontal and vertical directions. Its idealized representation and reactions are shown in Table 3.1.

3.2.2 Roller Support

A roller support allows rotation about any axis and translation (horizontal movement) in any direction parallel to the surface on which it rests. It restrains the structure from movement in a vertical direction. The idealized representation of a roller and its reaction are also shown in Table 3.1.

3.2.3 Rocker Support

The characteristics of a rocker support are like those of the roller support. Its idealized form is depicted in Table 3.1.

3.2.4 Link

A link has two hinges, one at each end. It permits movement in all direction, except in a direction parallel to its longitudinal axis, which passes through the two hinges. In other words, the reaction force of a link is in the direction of the link, along its longitudinal axis.

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3.2.5 Fixed Support

A fixed support offers a constraint against rotation in any direction, and it prevents movement in both horizontal and vertical directions.

3.3 Determinacy and Stability of Beams and Frames

Prior to the choice of an analytical method, it is important to establish the determinacy and stability of a structure. A determinate structure is one whose unknown external reaction or internal members can be determined using only the conditions of equilibrium. An indeterminate structure is one whose unknown forces cannot be determined by the conditions of static equilibrium alone and will require, in addition, a consideration of the compatibility conditions of different parts of the structure for its complete analysis. Furthermore, structures must be stable to be able to serve their desirable functions. A structure is considered stable if it maintains its geometrical shape when subjected to external forces.

3.3.1 Formulations for Stability and Determinacy of Beams and Frames

The conditions of determinacy, indeterminacy, and instability of beams and frames can be stated as follows:

$$3m + r < 3j + C \quad \text{Structure is statically unstable}$$

$$3m + r = 3j + C \quad \text{Structure is statically determinate}$$

$$3m + r > 3j + C \quad \text{Structure is statically indeterminate} \qquad (3.3)$$

where

r = number of support reactions.

C = equations of condition (two equations for one internal roller and one equation for each internal pin).

m = number of members.

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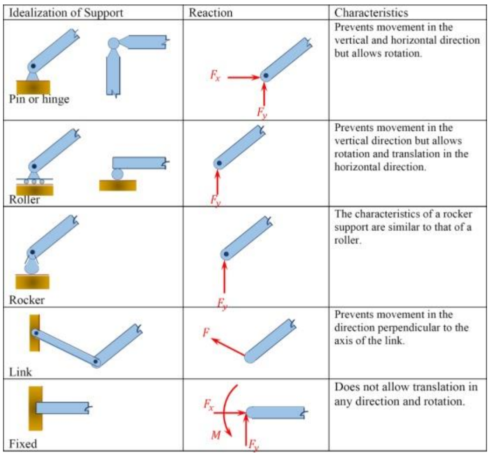
j = number of joints.

Table 3.1. Types of supports.



Idealization of Support	Reaction	Characteristics
Pin or hinge	F_x, F_y	Prevents movement in the vertical and horizontal direction but allows rotation.
Roller	F_y	Prevents movement in the vertical direction but allows rotation and translation in the horizontal direction.
Rocker	F_y	The characteristics of a rocker support are similar to that of a roller.
Link	F	Prevents movement in the direction perpendicular to the axis of the link.
Fixed	F_x, M, F_y	Does not allow translation in any direction and rotation.

3.3.2 Alternative Formulation for Determinacy and Stability of Beams and Frames

$$
\begin{aligned}
r + F_i &< 3m \quad \text{Structure is statically unstable} \\
r + F_i &= 3m \quad \text{Structure is statically determinate} \qquad (3.4) \\
r + F_i &> 3m \quad \text{Structure is statically indeterminate}
\end{aligned}
$$

where

r = number of support reactions.

F_i = number of reaction forces transmitted by an internal hinge or internal roller.

m = number of members.

Example 3.1

Classify the beams shown in Figure 3.1 through Figure 3.5 as stable, determinate, or indeterminate, and state the degree of indeterminacy where necessary.

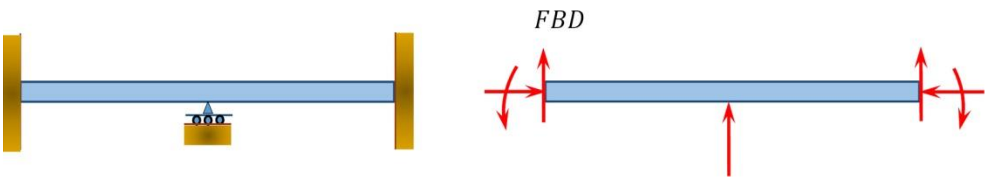


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Fig. 3.1. Beam.

Solution

First, draw the free-body diagram of each beam. To determine the classification, apply equation 3.3 or equation 3.4.

Using equation 3.3, $r = 7$, $m = 2$, $c = 0$, $j = 3$. Applying the equation leads to $3(2) + 7 > 3(3) + 0$, or $13 > 9$. Therefore, the beam is statically indeterminate to the 4°.

Using equation 3.4, $r = 7$, $m = 1$, $F_i = 0$. Applying the equation leads to $7 + 0 > (3)(1)$, or $7 > 3$. Therefore, the beam is statically indeterminate to the 4°.

Note: When using equation 3.3, the portions on either side of the interior support are counted as separate members.

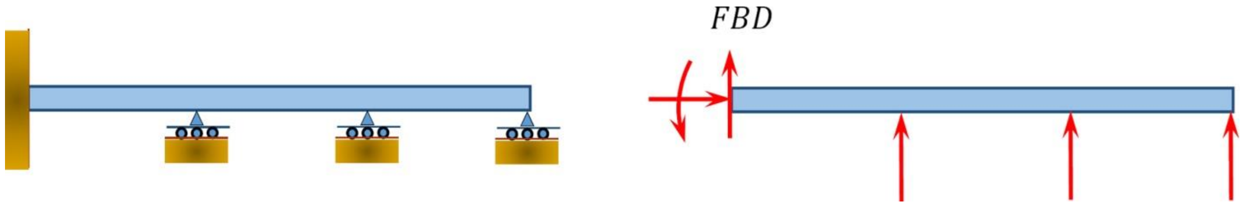

Fig. 3.2. Beam.

Solution

Using equation 3.3, $r = 6$, $m = 3$, $c = 0$, $j = 4$. Applying the equation leads to $3(3) + 6 > 3(4) + 0$, or $15 > 12$. Therefore, the beam is statically indeterminate to the 3°.

Using equation 3.4, $r = 6$, $m = 1$, $F_i = 0$. Applying the equation leads to $6 + 0 > (3)(1)$, or $6 > 3$. Therefore, the beam is statically indeterminate to the 3°.

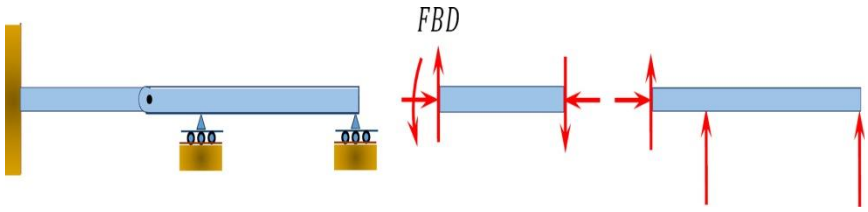

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Fig. 3.3. Beam.

Solution

Using equation 3.3, $r = 5$, $m = 3$, $c = 1$, $j = 4$. Applying the equation leads to 3(3) + 5 > 3(4) + 1, or 14 > 13. Therefore, the beam is statically indeterminate to the 1°.

Using equation 3.4, $r = 5$, $m = 2$, $F_i = 2$. Applying the equation leads to 5 + 2 > 3(2), or 7 > 6. Therefore, the beam is statically indeterminate to the 1°.

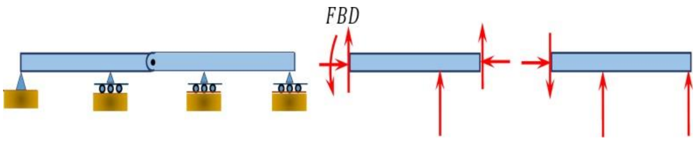

Fig. 3.4. Beam.

Solution

Using equation 3.3, $r = 5$, $m = 4$, $c = 1$, $j = 5$. Applying the equation leads to 3(4) + 5 > 3(5) + 1, or 17 > 16. Therefore, the equation is statically indeterminate to the 1°.

Using equation 3.4, $r = 5$, $m = 2$, $F_i = 2$. Applying the equation leads to 5 + 2 > 3(2), or 7 > 6. Therefore, the beam is statically indeterminate to the 1°.

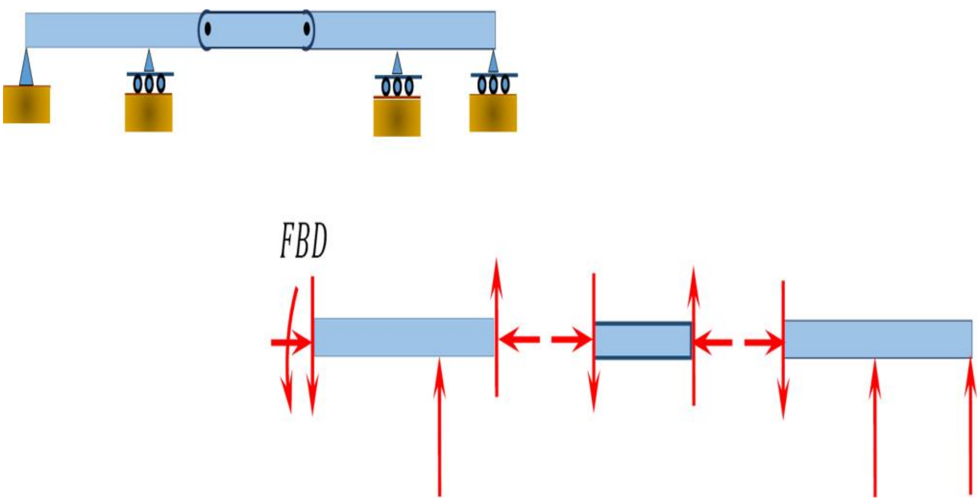

Fig. 3.5. Beam.

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Solution

Using equation 3.3, $r = 5$, $m = 5$, $c = 2$, $j = 6$. Applying the equation leads to 3(5) + 5 = 3(6) + 2, or 20 = 20. Therefore, the beam is statically determinate.

Using equation 3.4, $r = 5$, $m = 3$, $F_i = 4$. Applying the equation leads to 5 + 4 > 3(3), or 9 = 9. Therefore, the beam is statically determinate.

Example 3.2

Classify the frames shown in Figure 3.6 through Figure 3.8 as stable or unstable and determinate or indeterminate. If indeterminate, state the degree of indeterminacy.

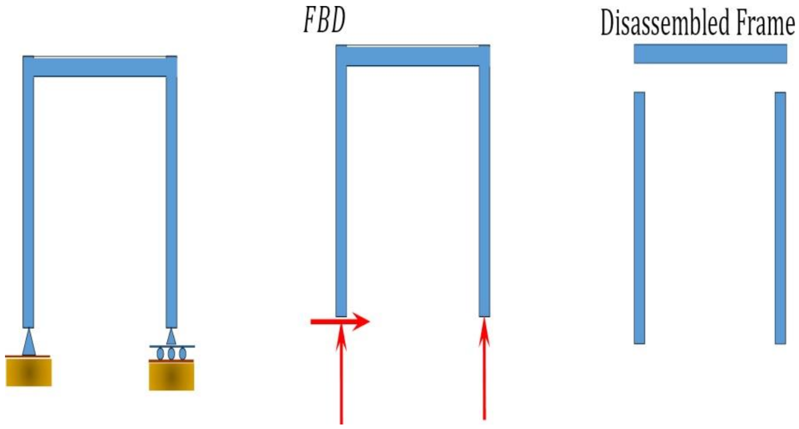

Fig. 3.6. Frame.

Solution

Using equation 3.3, $r = 3$, $m = 3$, $c = 0$, $j = 4$. Applying the equation leads to 3(3) + 3 = 3(4) + 0, or 12 = 12. Therefore, the frame is statically determinate.

Using equation 3.4, $r = 3$, $m = 1$, $F_i = 0$. Applying the equation leads to 3 + 0 = (3)(1), or 3 = 3. Therefore, the frame is statically determinate.

Note: When using equation 3.3 for classifying a frame, the frame must be disassembled at its joints to correctly determine the number of members.

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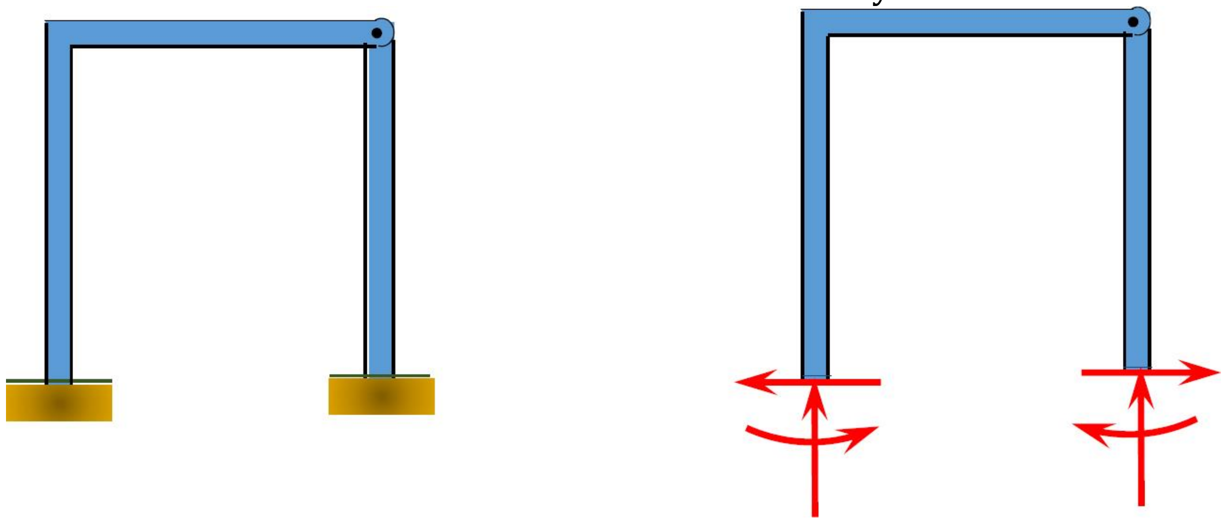

Fig. 3.7. Frame.

Solution

Using equation 3.3, $r = 6$, $m = 3$, $c = 1$, $j = 4$. Applying the equation leads to $3(3) + 6 > 3(4) + 1$, or $15 > 13$. Therefore, the frame is statically indeterminate to the 2°.

Using equation 3.4, $r = 6$, $m = 2$, $F_i = 2$. Applying the equation leads to $6 + 2 > 3(2)$, or $8 > 6$. Therefore, the frame is statically indeterminate to the 2°.

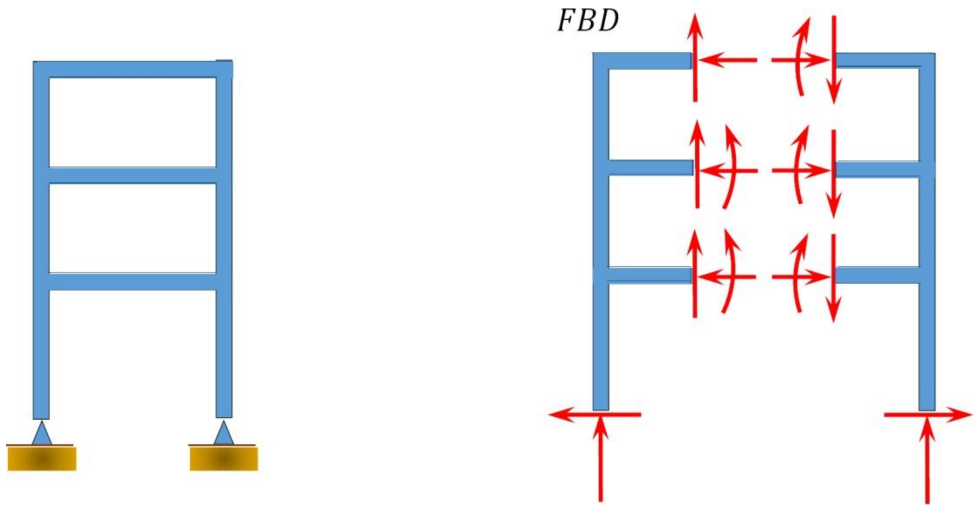

Fig. 3.8. Frame.

Solution

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Using equation 3.3, r = 4, m = 9, c = 0, j = 8. Applying the equation leads to 3(9) + 4 > 3(8) + 0, or 31 > 24. Therefore, the frame is statically indeterminate to the 7°.

Using equation 3.4, r = 4, m = 1, F_i = 9. Applying the equation leads to 4 + 9 > (3)(2), or 13 > 6. Therefore, the frame is statically indeterminate to the 7°.

Note: When using equation 3.4 to classify a frame with a closed loop, as given here, the loop has to be cut open by the method of section, and the internal reactions in the cut section should be considered in the analysis.

3.4 Computation of Support Reactions for Planar Structures

The support reactions for statically determinate and stable structures on a plane are determined by using the equations of equilibrium. The procedure for computation is outlined below.

Procedure for Computation of Support Reactions

•Sketch a free-body diagram of the structure, identifying all the unknown reactions using an arrow diagram.

•Check the stability and determinacy of the structure using equation 3.3 or 3.4. If the structure is classified as determinate, proceed with the analysis.

•Determine the unknown reactions by applying the three equations of equilibrium. If a computed reaction results in a negative answer, the initially assumed direction of the unknown reaction, as indicated by the arrow head on the free-body diagram, is wrong and should be corrected to show the opposite direction. Once the correction is made, the magnitude of the force should be indicated as a positive number in the corrected arrow head on the free-body diagram

Example 3.3

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A cantilever beam is subjected to a uniformly distributed load and an inclined concentrated load, as shown in figure 3.9a. Determine the reactions at support *A*.

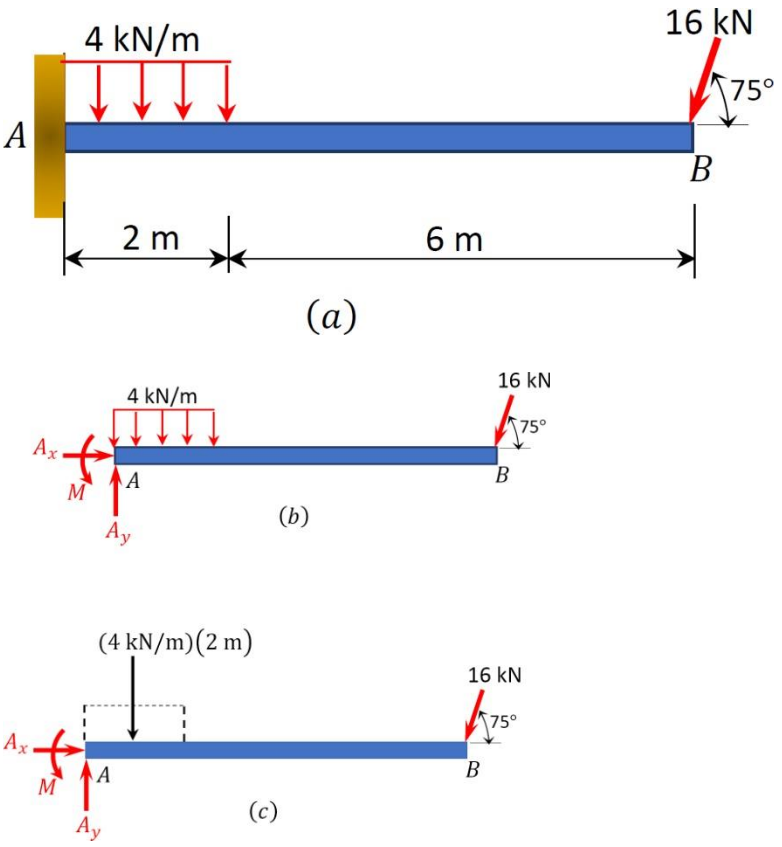

Fig. 3.9. Beam

Solution

Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.9b. The support reactions, as indicated in the free-body diagram, are A_y, A_x, and M.

Computation of reactions. Prior to the computation of the support reactions, the distributed loading should be replaced by a single resultant force, and the inclined loading resolved to the vertical and horizontal components. The magnitude of the resultant force is equal to the area under the rectangular loading, and it acts through the centroid of the rectangle. As seen in Figure 3.9c, P = [(4 kN/m)(2 m)], and its location is at the centroid of the rectangle

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loading $= \left[\left(\frac{1}{2}\right)(2\ \text{m})\right].$ Applying the equations of static equilibrium provides the following:

$$\curvearrowleft + \sum M_A = 0$$

$$-(16\sin 75°)(8) - (4 \times 2)(1) + M_A = 0$$

$$M_A = 131.64\ \text{kN.m} \qquad M_A = 131.64\ \text{kN.m}$$

$$\uparrow + \sum F_y = 0$$

$$A_y - 16\sin 75° - (4 \times 2) = 0$$

$$A_y = 23.45\ \text{kN} \qquad A_y = 23.45\ \text{kN} \uparrow$$

$$\rightarrow + \sum F_x = 0$$

$$A_x = 0 \qquad A_x = 0$$

Example 3.4

A 12ft-long simple beam carries a uniformly distributed load of 2 kips/ft over its entire span and a concentrated load of 8 kips at its midspan, as shown in Figure 3.10a. Determine the reactions at the supports *A* and *B* of the beam.

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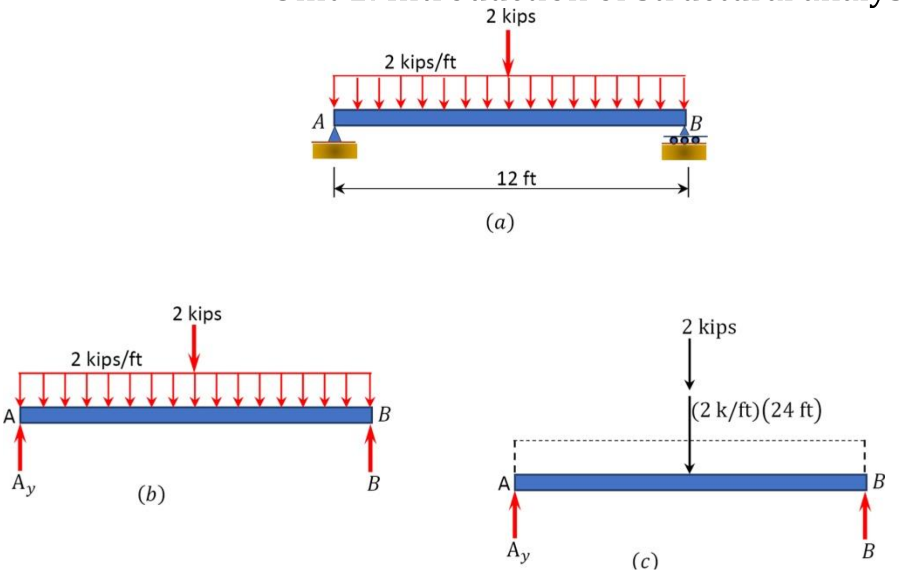

Fig. 3.10. Simple beam.

Solution

Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.10b.

Computation of reactions. The distributed loading is first replaced with a single resultant force, as seen in Figure 3.10c. The magnitude of the resultant force is equal to the area of the rectangular loading (distributed force). Thus, *P* = [(2 k/ft)(12 ft)], and its location is at the centroid of the rectangular loading $= \left[\left(\frac{1}{2}\right)(12\text{ft})\right]$. Since there is a symmetry in loading in this example, the reactions at both ends of the beam are equal, and they could be determined using the equations of static equilibrium and the principle of superposition, as follows:

$$+\uparrow \sum F_y = 0$$

$$A_y = B_y = \left(\frac{2\times12}{2}\right) + \frac{2}{2} = 13 \text{ kips} \qquad A_y = B_y = 13 \text{ kips} \uparrow$$

$$+\rightarrow \sum \mathrm{F_x} = 0$$

$$\mathrm{A_x} = 0 \qquad \mathrm{A_x} = 0$$

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Example 3.5

A beam with an overhang is subjected to a varying load, as shown in Figure 3.11a. Determine the reactions at supports *A* and *B*.

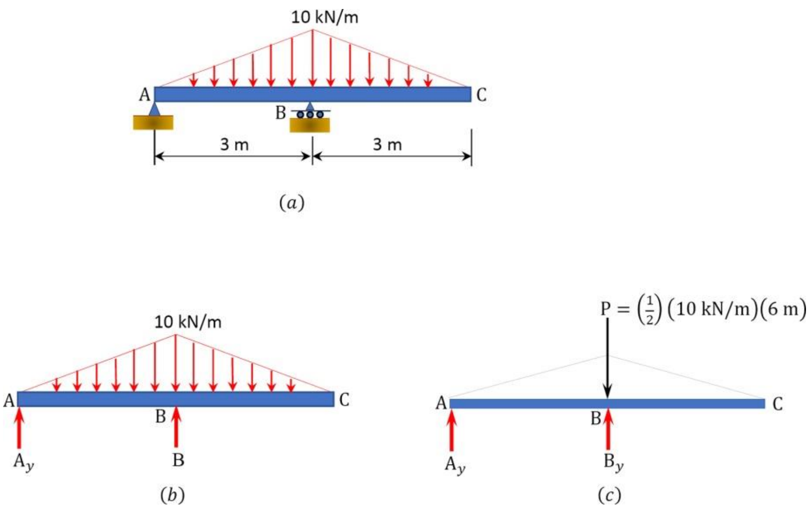

Fig. 3.11. Beam with an overhang.

Solution

Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.11b.

Computation of reactions. Observe that the distributed loading in the beam is triangular. The distributed load is first replaced with a single resultant force, as shown in Figure 3.11c. The magnitude of the single resultant force is equal to the area under the triangular loading. Thus, $P = \left(\frac{1}{2}\right)(6\ \text{m})(10\ \text{kN/m})$, and its centroid is at the center of the loading (6m). Applying the equations of equilibrium provides the following:

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$$\curvearrowleft + \sum M_A = 0$$

$$-\left(\frac{1}{2}\right)(10)(6)(3) + 3B = 0$$

$B_y = 30 \text{ kN}$ $\qquad B_y = 30 \text{ kN} \uparrow$

$$\uparrow + \sum F_y = 0$$

$30 + A_y - \left(\frac{1}{2}\right)(6)(10) = 0$ $\qquad A_y = 0$

$$\rightarrow + \sum F_x = 0$$

$A_x = 0$ $\qquad A_x = 0$

Example 3.6

A beam with overhanging ends supports three concentrated loads of 12 kips, 14 kips, and 16 kips and a moment of 100 kips.ft, as shown in Figure 3.12a. Determine the reactions at supports *A* and *B*.

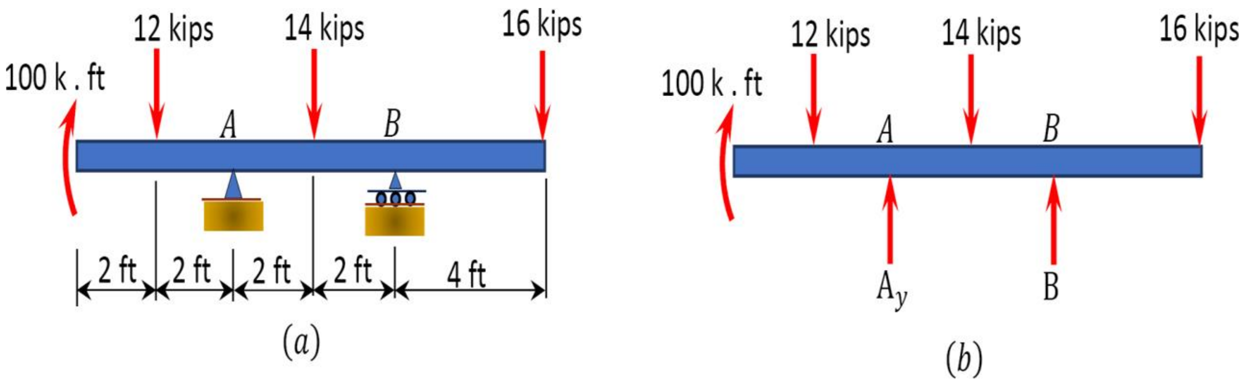

Fig. 3.12. Beam with overhanging ends.

Solution

Free-body diagram. The free-body diagram of the beam is shown in Figure 3.12b.

Computation of reactions. Applying the equations of equilibrium provides the following:

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$$+\curvearrowleft \sum M_A = 0$$

$$-100 + 12(2) - 14(2) - 16(8) + 4B_y = 0$$

$B_y = 58 \text{ kips}$ $\qquad B_y = 58 \text{ kips} \uparrow$

$$+\uparrow \sum F_y = 0$$

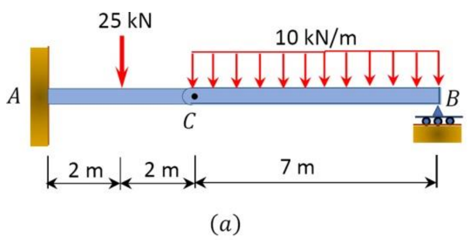
$58 + A_y - 12 - 14 - 16 = 0$ $\qquad A_y = 16 \text{ kips} \uparrow$

$$+\rightarrow \sum F_x = 0$$

$A_x = 0$ $\qquad A_x = 0$

Example 3.7

A compound beam is subjected to the loads shown in Figure 3.13a. Find the support reactions at *A* and *B* of the beam.



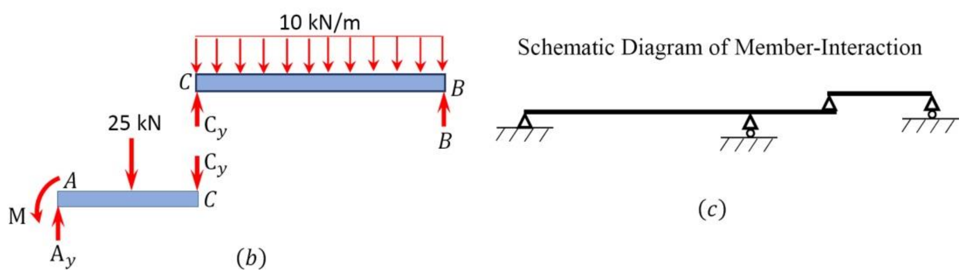


Fig. 3.13. Compound beam.

Solution

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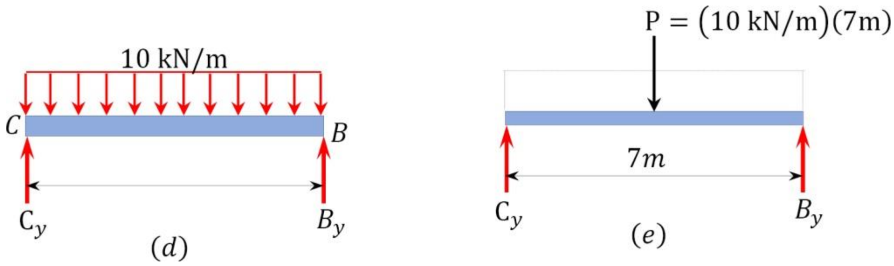
Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.13b.

Identification of primary and complimentary structures. For correct analysis of a compound structure, the primary and the complimentary parts of the structure should be identified for proper understanding of their interaction. The interaction of these parts are shown in Figure 3.13c. The primary structure is the part of the compound structure that can sustain the applied external load without the assistance of the complimentary structure. On the other hand, the complimentary structure is the part of the compound structure that depends on the primary structure to support the applied external load. For the given structure, part *AC* is the primary structure, while part *CB* is the complimentary structure.

Computation of reactions. The analysis of a compound structure must always begin with the analysis of the complimentary structure, as the complimentary structure is supported by the primary structure. Using the equations of equilibrium, the support reactions of the beam are determined as follows:

Analysis of the complimentary structure *CB*.



Computation of support reaction. The isolated free-body diagram of the complimentary structure is shown in Figure 3.13c. First, the distributed loading is replaced by a single resultant force (*P*), which is equal to the area of the rectangular loading, as shown in Figure 3.13d and Figure 3.13e. Applying the equations of equilibrium, and noting that due to symmetry in loading, the support reactions at point *C* and *B* are equal in magnitude, provides the following:

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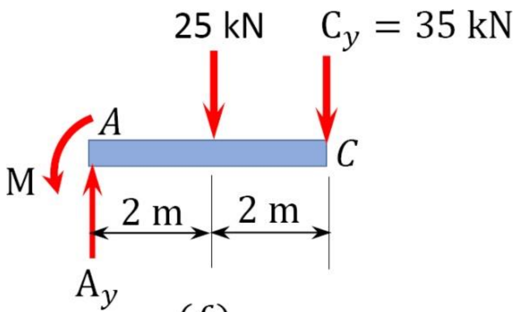
$$+\uparrow \sum F_y = 0$$

$$B_y = C_y = \frac{10(7)}{2} = 35\text{ kN}$$

Analysis of the primary structure *AC*.



(*f*)

Computation of support reaction. Note that prior to the computation of the reactions, the reaction at point *C* in the complimentary structure is applied to the primary structure as a load. The magnitude of the applied load is the same as that of the complimentary structure, but it is opposite in direction. Applying the equations of equilibrium suggests the following:

$$+\curvearrowleft \sum M_A = 0$$

$$-25(2) - 35(4) + M_A = 0$$
$$M_A = 190\text{ kN.m} \qquad M_A = 190\text{ kN.m} \curvearrowleft$$

$$+\uparrow \sum F_y = 0$$
$$A_y - 25 - 35 = 0$$
$$A_y = 60\text{ kN} \qquad A_y = 60\text{ kN}\uparrow$$

$$+\rightarrow \sum F_x = 0$$
$$A_x = 0 \qquad A_x = 0$$

Example 3.8

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Find the reactions at supports *A*, *C*, and *E* of the compound beam carrying a uniformly distributed load of 10 kips/ft over its entire length as shown in figure 3.14a.

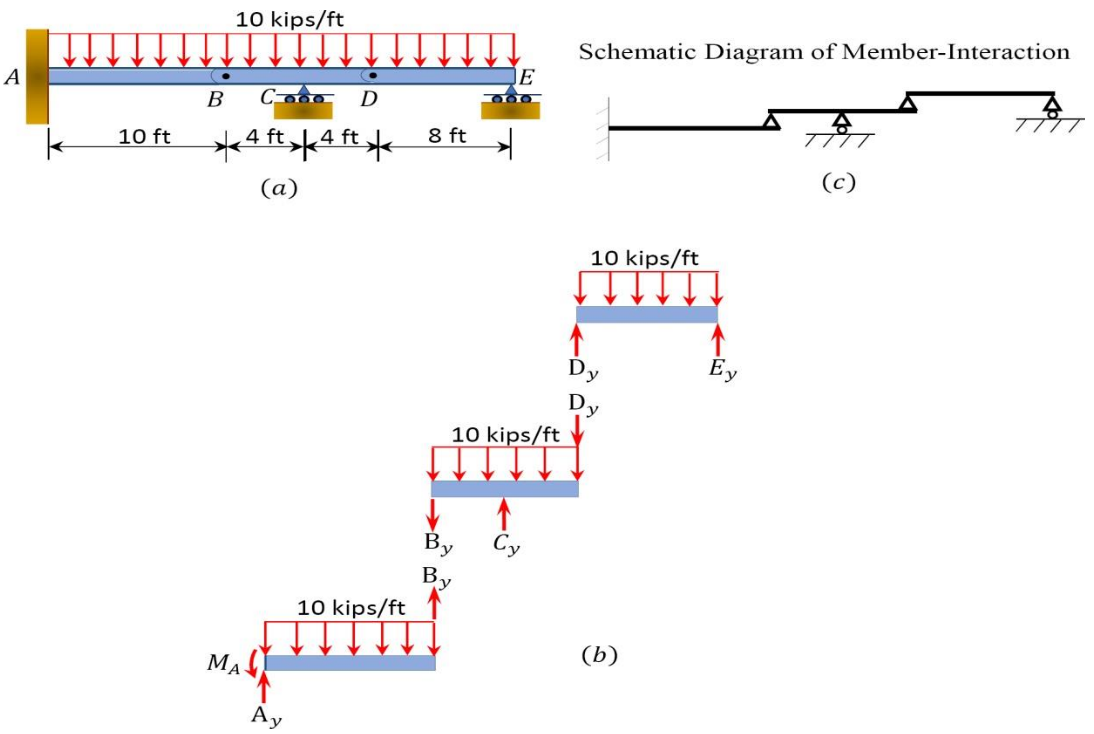

Fig. 3.14. Compound beam.

Solution

Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.14b.

Identification of primary and complimentary structures. The interaction diagram for the given structure is shown in Figure 3.14c. *AB* is the primary structure, while *BD* and *DE* are the complimentary structures.

Computation of reactions.

Analysis of complimentary structure *DE*.

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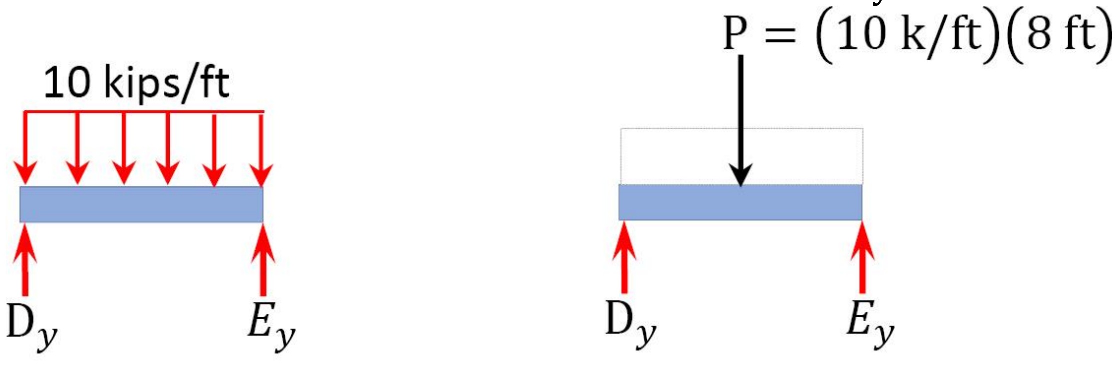


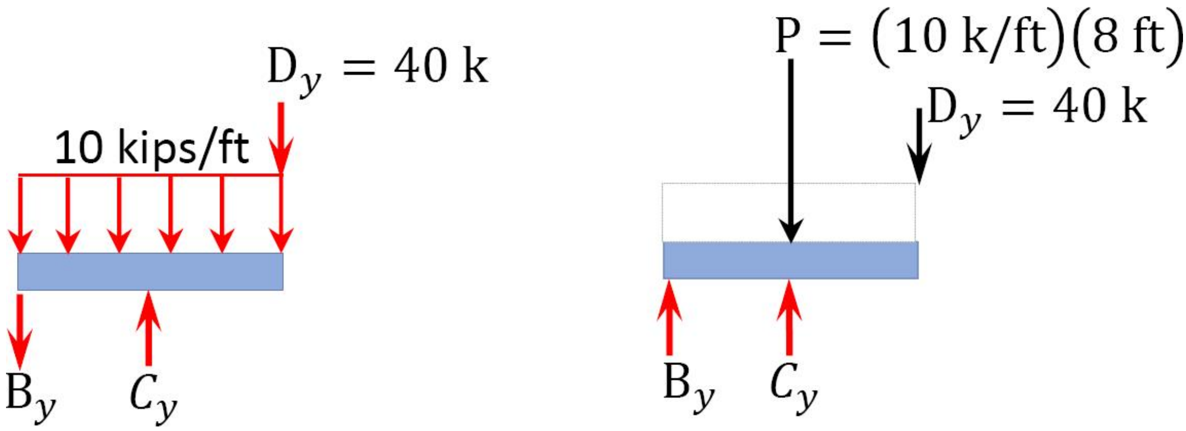
(c) (d)

Computation of support reaction. The isolated free-body diagram is shown in Figure 3.14c. First, the distributed loading is replaced by a single resultant force (P) equal the area of rectangular loading, as shown in Figure 3.14d. Applying the equations of equilibrium, and noting that due to symmetry in loading, the support reactions at point D and E are equal in magnitude, suggests the following:

$$+\uparrow \sum F_y = 0$$

$$D_y = E_y = \frac{10(8)}{2} = 40 \text{ kips} \qquad E_y = 40 \text{ kips} \uparrow$$

Analysis of complimentary structure BD.



(e) (f)

Computation of support reaction. The isolated free-body diagram is shown in Figure 3.14e. First, the distributed loading is replaced by a single resultant force (P) equal to the area of the rectangular loading, as shown in Figure 3.14f. The load from the complimentary structure

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is applied at point *D*. Applying the equations of equilibrium suggests the following:

$$+\curvearrowleft \sum M_B = 0$$

$$-10(8)\left(\frac{8}{2}\right) - 40(8) + 4C_y = 0$$

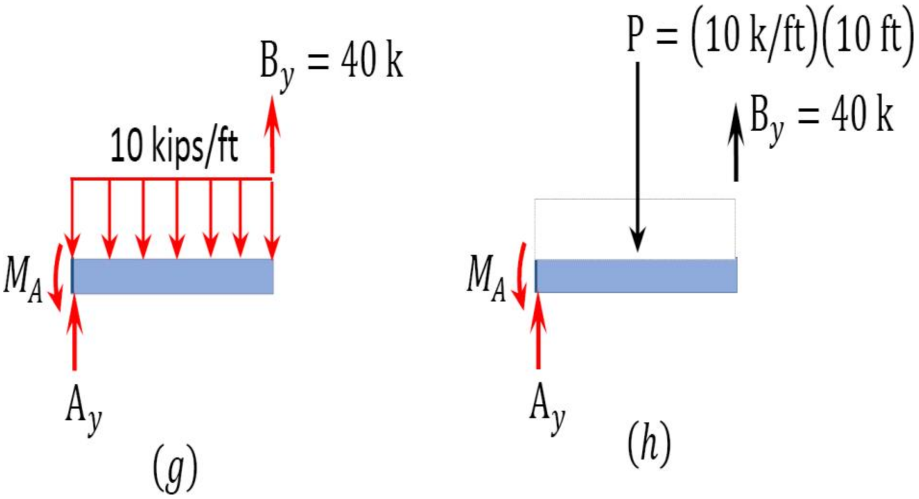
$C_y = 160$ kips $\qquad\qquad C_y = 160$ kips ↑

$$+\uparrow \sum F_y = 0$$

$$160 - B_y - 10(8) - 40 = 0$$

$$B_y = 40 \text{ kips}$$

Analysis of primary structure *AB*.



Computation of support reaction. Note that prior to the computation of the reactions, the uniform load is replaced by a single resultant force, and the reaction at point *B* in the complimentary structure is applied to the primary structure as a load. Applying the equilibrium requirement yields the following:

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$+\curvearrowleft \sum M_A = 0$

$M - 10(10)\left(\frac{10}{2}\right) + 40(10) = 0$

$M_A = 100 \text{ kips.ft}$ $\quad\quad M_A = 100 \text{ kips.ft} \curvearrowleft$

$+\uparrow \sum F_y = 0$

$A_y - 10(10) + 40 = 0$

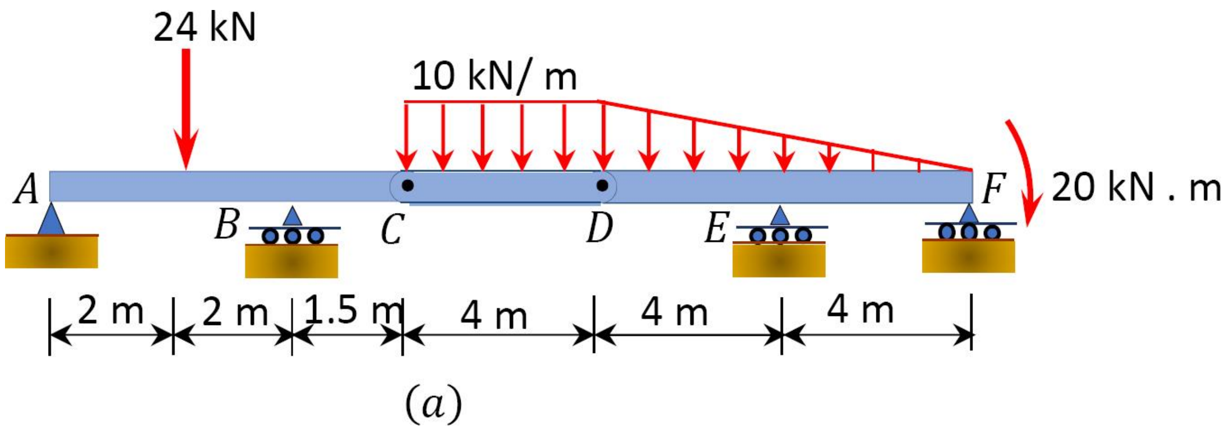
$A_y = 60 \text{ kips}$ $\quad\quad A_y = 60 \text{ kips} \uparrow$

$+\rightarrow \sum F_x = 0$

$A_x = 0$ $\quad\quad A_x = 0$

Example 3.9

Find the reactions at supports *A*, *B*, *E*, and *F* of the loaded compound beam, as shown in Figure 3.15a.



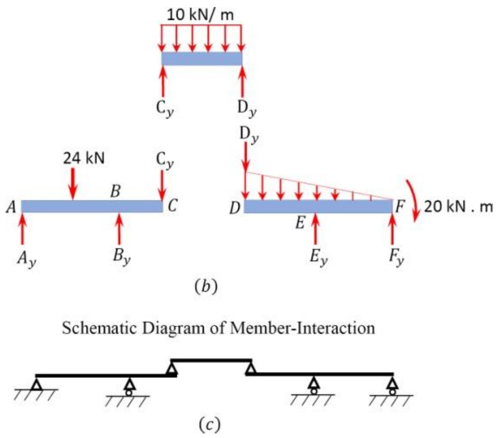


Fig. 3.15. Compound beam.

Solution

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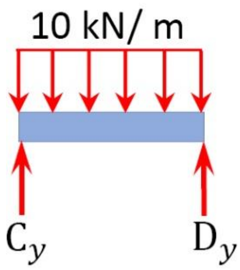
Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.15b.

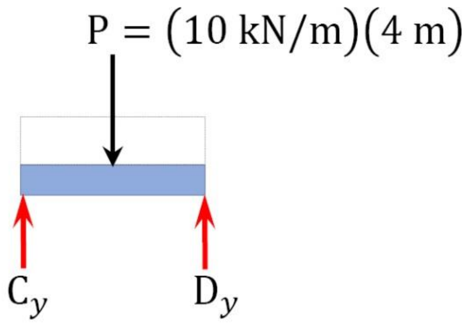
Identification of primary and complimentary structure. The interaction diagram for the given structure is shown in Figure 3.15c. *CD* is the complimentary structure, while *AC* and *DF* are the primary structures.

Computation of reactions.

Analysis of complimentary structure *CD*.



(c)



(d)

Computation of support reaction. The isolated free-body diagram is shown in Figure 3.15c. First, the distributed loading is replaced by a single resultant force (*P*), which is equal to the area of the rectangular loading, as shown in Figure 3.15d. Applying the equations of equilibrium, and noting that due to symmetry in loading, the support reactions at point *C* and *D* are equal in magnitude, suggests the following:

$$+\uparrow \sum F_y = 0$$

$$C_y = D_y = \frac{10(4)}{2} = 20 \text{ kN}$$

Analysis of primary structure *AC*.

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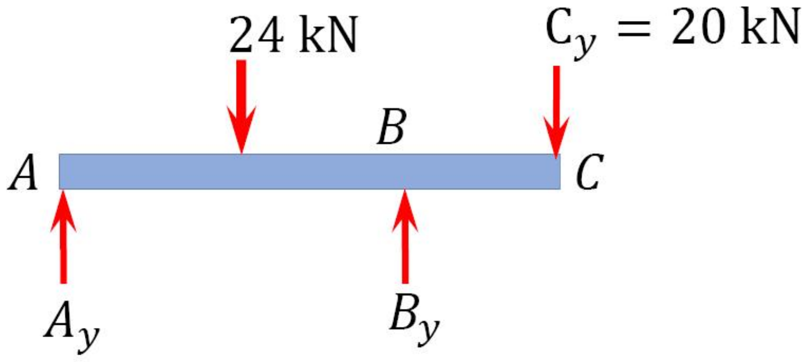

(e)

Computation of support reaction. Note that the reaction at *C* of the complimentary structure is applied as a downward force of the same magnitude at the same point on the primary structure. Applying the equation of equilibrium suggests the following:

$$+\curvearrowleft \sum M_A = 0$$

$$-24(2) - 20(5.5) + 4B_y = 0$$

$B_y = 39.5$ kN $\qquad\qquad B_y = 39.5$ kN $\uparrow$

$$+\uparrow \sum F_y = 0$$

$$A_y + 39.5 - 24 - 20 = 0$$

$A_y = 4.5$ kN $\qquad\qquad A_y = 4.5$ kN $\uparrow$

$$+\rightarrow \sum F_x = 0$$

$A_x = 0 \qquad\qquad A_x = 0$

Analysis of primary structure *DF*.

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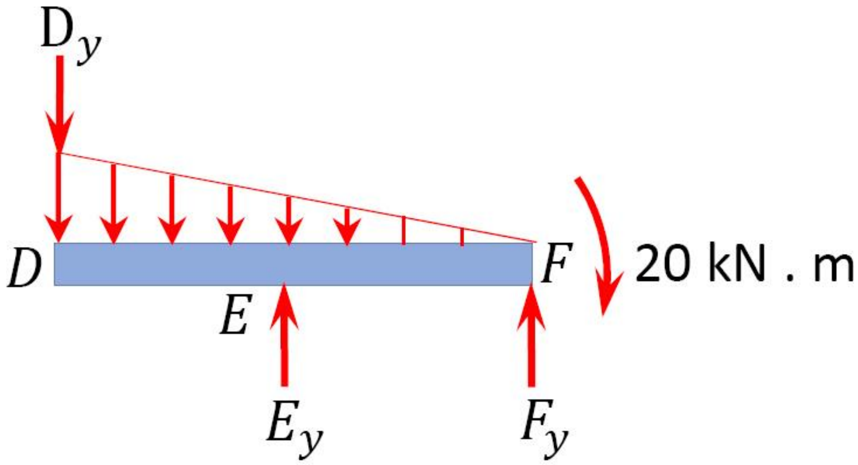
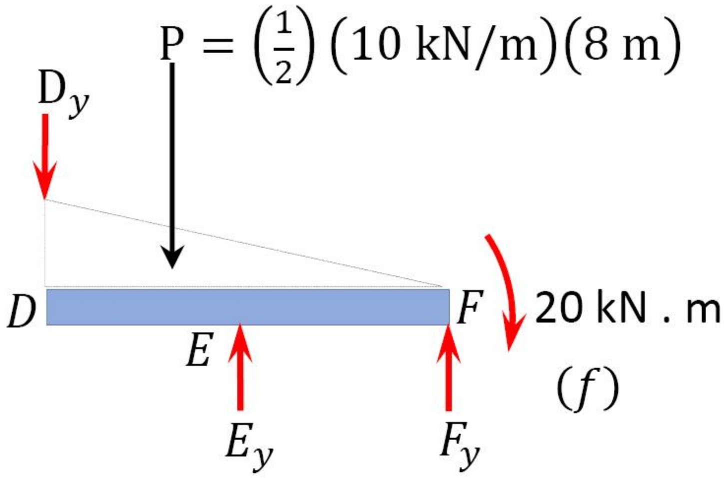


(f)



(g)

Computation of support reaction. The isolated free-body diagram is shown in Figure 3.15f. First, the distributed loading is replaced by a single resultant force (*P*) equal to the area of the triangular loading, as shown in Figure 3.15g. Applying the equations of equilibrium, and noting that the support reaction at point *D* of the complimentary structure is applied as a load on the primary structure, suggests the following:

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$$+\circlearrowleft \sum M_F = 0$$

$$-20 + \left(\frac{1}{2} \times 8 \times 10\right)\left(\frac{2}{3} \times 8\right) + 20(8) - 4E_y = 0$$

$$E_y = 88.33 \text{ kN} \qquad E_y = 88.33\text{kN} \uparrow$$

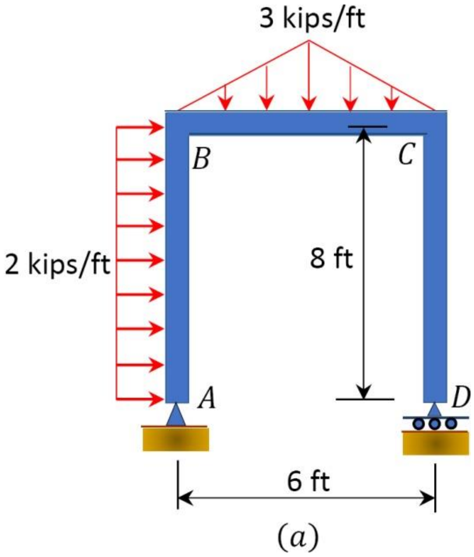
$$+\uparrow \sum F_y = 0$$

$$F_y + 88.33 - \left(\frac{1}{2} \times 8 \times 10\right) - 20 = 0$$

$$F_y = 28.33 \text{ kN} \qquad F_y = 28.33 \text{ kN} \uparrow$$

Example 3.10

Determine the reactions at supports *A* and *D* of the frame shown in Figure 3.16a.



(a)

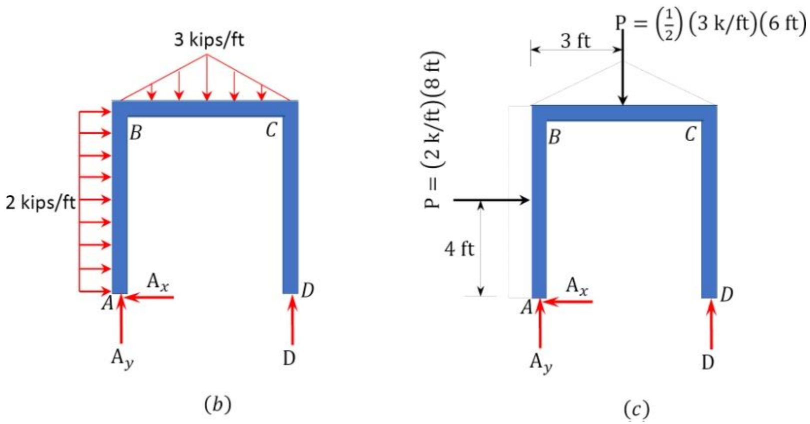


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Fig. 3.16. Frame.

Solution

Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.16b.

Computation of reactions. The distributed loads in column *AB* and beam *BC* are first replaced by single resultant forces determined as the area of their respective shade of loading, as shown in Figure 3.16c. Applying the conditions of equilibrium suggests the following:

$$+\curvearrowleft \sum M_A = 0$$

$$D_y(6) - \left(\frac{1}{2}\right)(6)(3)(3) - (2)(8)(4) = 0$$

$D_y = 15.7 \text{ kips}$ $\qquad D_y = 15.7 \text{ kips} \uparrow$

$$+\uparrow \sum F_y = 0$$

$$A_y + 15.17 - 3(6) = 0$$

$A_y = 2.830 \text{ kips}$ $\qquad A_y = 2.830 \text{ kips} \uparrow$

$$+\rightarrow \sum F_x = 0$$

$$-A_x + (2 \times 8) = 0$$

$A_x = 16 \text{ kips}$ $\qquad A_x = 16 \text{ kips} \leftarrow$

Example 3.11

A rigid frame is loaded as shown in Figure 3.17a. Determine the reactions at support *D*.

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Fig. 3.17. Rigid frame.

Solution

Free-body diagram. The free-body diagram of the entire beam is shown in Figure 3.17b.

Computation of reactions. The distributed load in portion *AB* of the frame is first replaced with a single resultant force, as shown in Figure 3.17c. Applying the equations of equilibrium suggests the following:

$+\curvearrowleft \sum M_D = 0$

$-M_D - 16(8) + (4 \times 14)\left(\frac{14}{2}\right) - 10(10) = 0$

$M_D = 164 \text{ kips.ft}$ $\qquad M_D = 164 \text{ kips.ft} \curvearrowright$ A

$+\uparrow \sum F_y = 0$

$D_y - 4(14) - 10 = 0$

$D_y = 66 \text{ kips}$ $\qquad D_y = 66 \text{ kips} \uparrow$

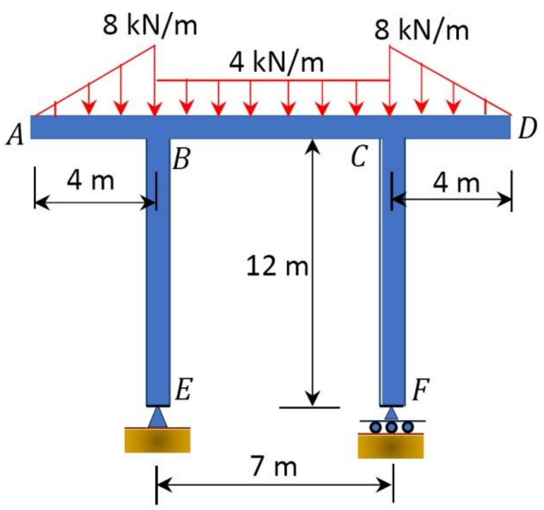
$+\rightarrow \sum F_x = 0$

$-D_x + 16 = 0$

$D_x = 16 \text{ kips}$ $\qquad D_x = 16 \text{ kips} \leftarrow$

Example 3.12

Find the reactions at supports *E* and *F* of the frame shown in Figure 3.18a.



(*a*)

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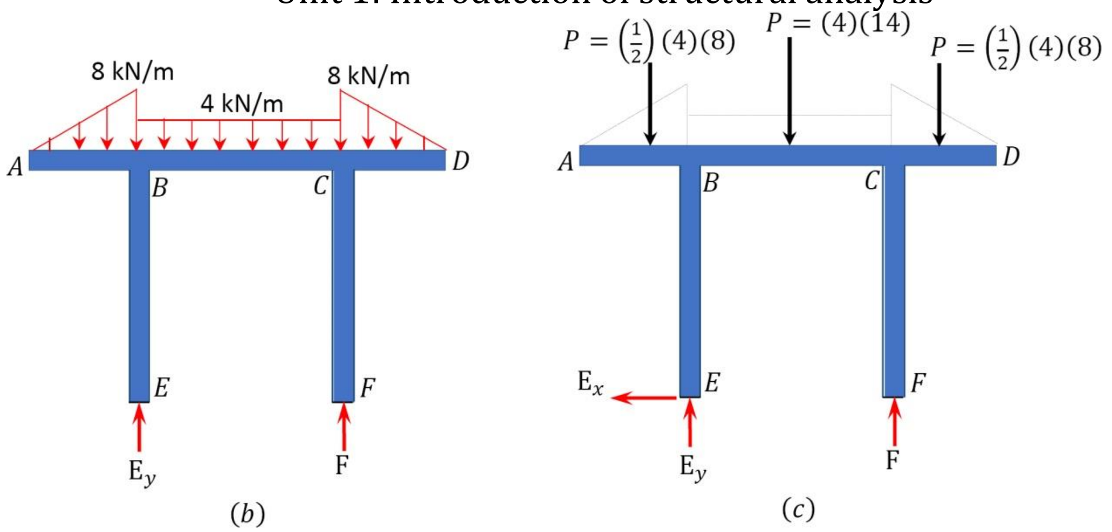

Fig. 3.18. Frame.

Solution

Free-body diagram. The free-body diagram of the frame is shown in Figure 3.18b.

Computation of reactions. The distributed loads are first replaced with single resultant forces, as shown in Figure 3.18c. Applying the equations of static equilibrium suggests the following:

$$+\curvearrowleft \sum M_E = 0$$

$$\left(\tfrac{1}{2}\times 4\times 8\right)\left(\tfrac{1}{3}\times 4\right) - (4\times 7)\left(\tfrac{7}{2}\right) - \left(\tfrac{1}{2}\times 4\times 8\right)\left(\tfrac{7}{2}+\tfrac{1}{3}\times 4\right) + 7F_y = 0$$

$F_y = 22$ kN $\qquad F_y = 22$ kN $\uparrow$

$$+\uparrow \sum F_y = 0$$

$$E_y + 22 - 2\left(\tfrac{1}{2}\times 4\times 8\right) - 4(7) = 0$$

$E_y = 38$ kN $\qquad E_y = 38$ kN $\uparrow$

$$+\rightarrow \sum F_x = 0$$

$E_x = 0 \qquad E_x = 0$

Example 3.13

Determine the reactions at support A of the rigid frame shown in Figure 3.19a.

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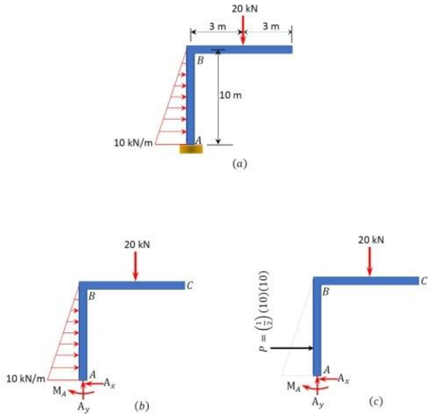

Fig. 3.19. Rigid frame.

Solution

Free-body diagram. The free-body diagram of the frame is shown in Figure 3.19b.

Computation of reactions. The distributed load in column *AB* is first replaced with a single resultant force, as shown in Figure 3.19c. Applying the equations of static equilibrium suggests the following:

$$+\curvearrowleft \sum M_A = 0$$

$$-M_A - 20(3) - \left(\frac{1}{2} \times 10 \times 10\right)\left(\frac{1}{3} \times 10\right) = 0$$

$M_A = -226.67 \text{ kN.m} \qquad M_A = 226.67 \text{ kN.m} \curvearrowleft$

$$+\uparrow \sum F_y = 0$$

$A_y - 20 = 0$

$A_y = 20 \text{ kN} \qquad A_y = 20 \text{ kN} \uparrow$

$+\rightarrow \sum F_x = 0$

$-A_x + \left(\frac{1}{2} \times 10 \times 10\right) = 0$

$A_x = 50 \text{ kN} \qquad A_x = 50 \text{ kN} \leftarrow$

Example 3.14

Determine the reactions at supports *A* and *E* of the frame hinged at *C*, as shown in Figure 3.20a.

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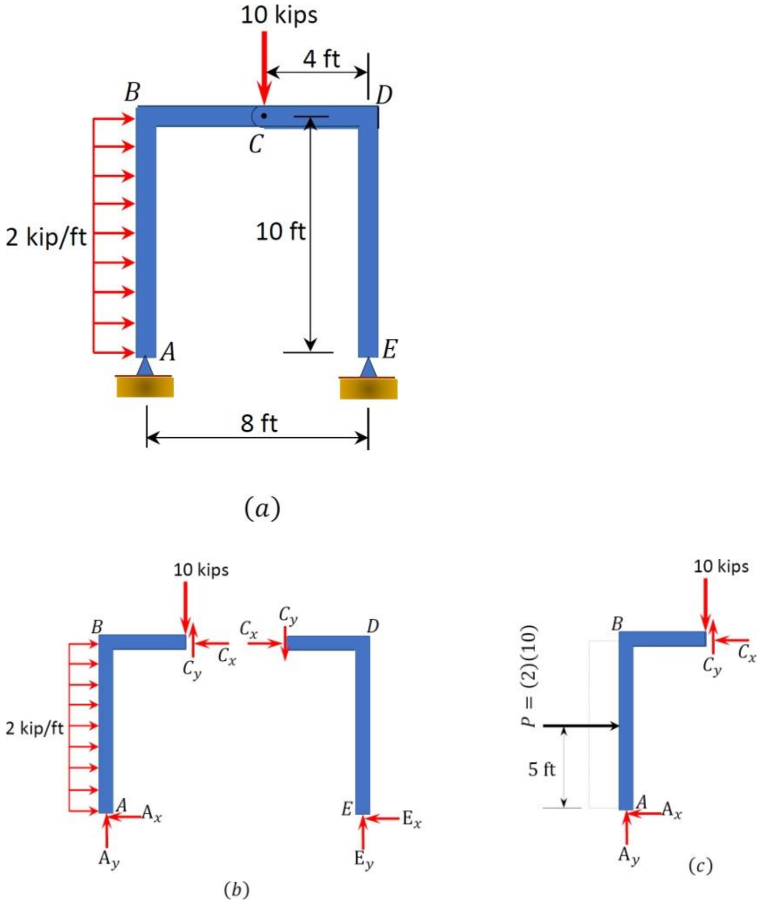

Fig. 3.20. Frame.

Solution

Free-body diagram. The free-body diagram of the frame is shown in Figure 3.20b.

Computation of reactions. The reactions in a compound frame are computed considering the freebody diagrams of both the entire frame and part of the frame. Prior to computation of the reactions, the distributed load in the column is replaced by a single resultant force. The vertical reactions at *E* and *A* and the horizontal reactions at *A* are found by applying the equations of static equilibrium and considering the free-body diagram of the entire frame. The horizontal reaction at *E* is found by considering part *CDE* of the free-body diagram.

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$+\curvearrowleft \sum M_A = 0$

$8E_y - (2 \times 10)\left(\frac{10}{2}\right) - 10(4) = 0$

$E_y = 17.5 \text{ kips}$ $E_y = 17.5 \text{ kips} \uparrow$

$+\uparrow \sum F_y = 0$

$A_y + 17.5 - 10 = 0$

$A_y = -7.5 \text{ kips}$ $A_y = 7.5 \text{ kips} \downarrow$

The negative sign implies that the originally assumed direction of A_y was not correct. Therefore, A_y acts downward instead of upward as was initially assumed. This should be corrected in the subsequent analysis.

To determine E_x, consider the moment of forces in member *CDE* about the hinge.

$$\curvearrowleft + \sum M_C = 0$$

$17.5(4) - 10E_x = 0$

$E_x = 7 \text{ kips}$ $E_x = 7\text{kips} \leftarrow$

$+ \rightarrow \sum F_x = 0$

$-A_x - 7 + 2 \times 10 = 0$

$A_x = 13 \text{ kips}$ $A_x = 13 \text{ kips} \leftarrow$

Example 3.15

Find the reactions at support *A* and *B* of the loaded frame in Figure 3.21a. The frame is hinged at *D*.

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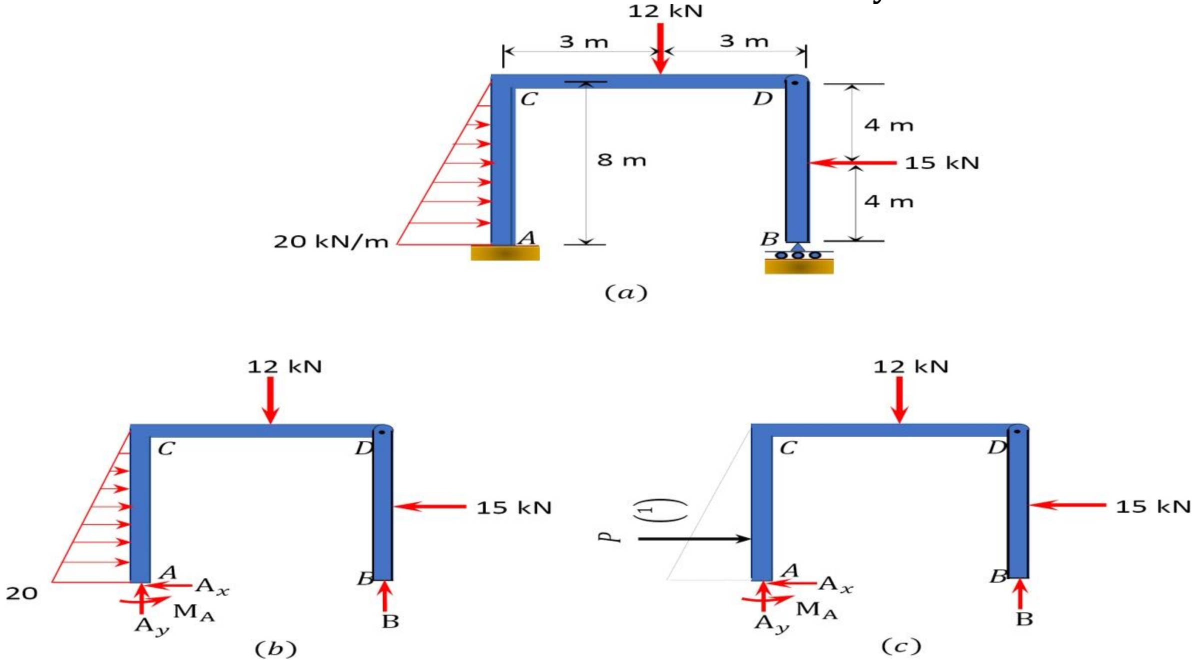

Fig. 3.21. Loaded frame.

Solution

Free-body diagram. The free-body diagram of the frame is shown in Figure 3.21b.

Computation of reactions. The distributed load in column *AC* is first replaced with a single resultant force by finding the area of loading, as shown in Figure 3.21Figurec. The reaction at *B* is computed by taking the moment of the forces in part *DB* of the frame about the pin at *D*, and other reactions are determined by applying other conditions of equilibrium.

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$$+\curvearrowleft \sum M_D = 0$$

$$B_y(0) - 15(4) = 0$$
$$B_y = 0$$
$$+\curvearrowleft \sum M_A = 0$$

$$M_A + 6 \times 0 - \left(\tfrac{1}{2} \times 8 \times 20\right)\left(\tfrac{1}{3} \times 8\right) - 12(3) + 15(4) = 0$$

$$M_A = 189.33 \text{ kN. m} \qquad M_A = 189.33 \text{ kN. m} \curvearrowright$$

$$+\uparrow \sum F_y = 0$$

$$A_y + 0 - 12 = 0$$
$$A_y = -12 \text{ kN} \qquad A_y = 12 \downarrow$$

The negative sign implies that the originally assumed direction of A_y was not correct. Therefore, A_y acts downward instead of upward as was initially assumed. This should be corrected in the subsequent analysis.

$$+\rightarrow \sum F_x = 0$$
$$-A_x - 15 + \left(\tfrac{1}{2} \times 8 \times 20\right) = 0$$
$$A_x = 65 \text{ kN} \qquad A_x = 65 \text{ kN} \rightarrow$$

Chapter Summary

Conditions of static equilibrium: A structure is in a state of static equilibrium if the resultant of all the forces and moments acting on it is equal to zero. Mathematically, this is expressed as follows:

$\sum F = 0 \sum M = 0$

For a body in a plane, there are the following three equations of equilibrium:

$\sum F_x = 0 \sum F_y = 0 \sum M_0 = 0$

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Types of support: Various symbolic representations are used to model different types of supports for structures. A roller is used to model a support that prevents a vertical movement of a structure but allows a horizontal translation and rotation. A pin is used to model a support that prevents horizontal and vertical movements but allows rotation. A fixed support models a support that prevents horizontal and vertical movements and rotation.

$V = 0$ $H \neq 0$ $\theta \neq 0$	$V = 0$ $H = 0$ $\theta \neq 0$	$V = 0$ $H = 0$ $\theta = 0$

Determinacy, indeterminacy, and stability of structures: A structure is determinate if the number of unknown reactions is equal to the number of static equilibrium. Thus, the equations of static equilibrium are enough for the determination of the supports for such a structure. On the other hand, a statically indeterminate structure is a structure that has the number of the unknown reactions in excess of the equations of equilibrium. For the analysis of an indeterminate structure additional equations are needed, and these equations can be obtained by considering the compatibility of the structure. Indeterminate structures are sometimes necessary when there is a need to reduce the sizes of members or to increase the stiffness of members. A stable structure is one which has support reactions that are not parallel or concurrent to one another. The formulation of stability and determinacy of beams and frames are as follows:

Beams and frames:	$3m + r < 3j + C$ Structure is
	$3m + r = 3j + C$ Structure is

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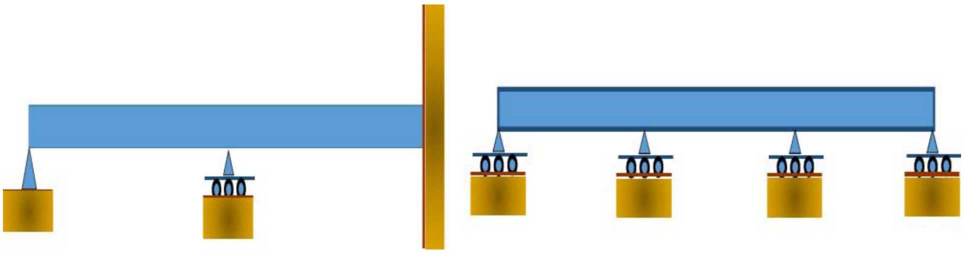
$3m + r > 3j + C$ Structure is

Practice Problems

3.1 Classify the structures shown in Figure P3.1a to Figure P3.1p as statically determinate or indeterminate, and statically stable or unstable. If indeterminate, state the degree of indeterminacy.



(a) (b)

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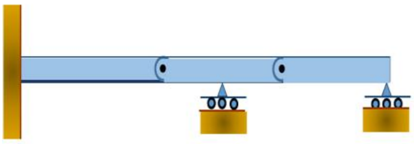
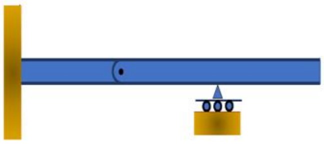
(c)

(d)



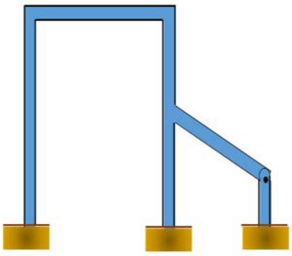
(e)



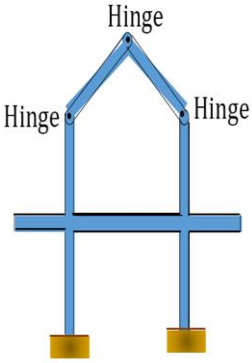
(f)

(g)

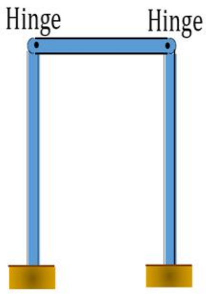
(h)



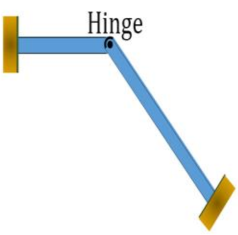
(i)



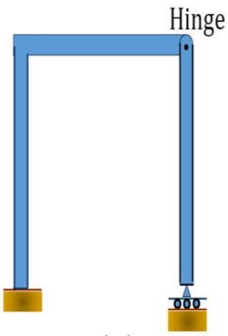
(j)



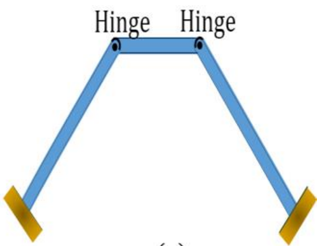
(k)



(l)



(m)



(n)

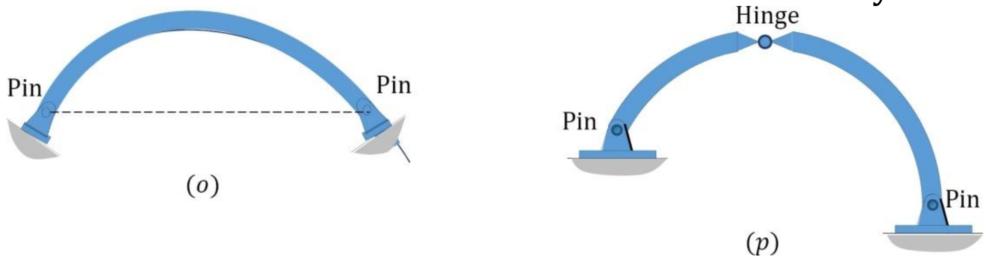


Fig. P3.1. Structure classification.

3.2. Determine the support reactions for the beams shown in Figure P3.2 through Figure P3.12.

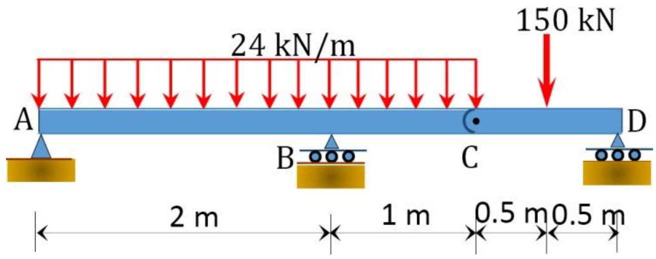


Fig. P3.2. Beam.

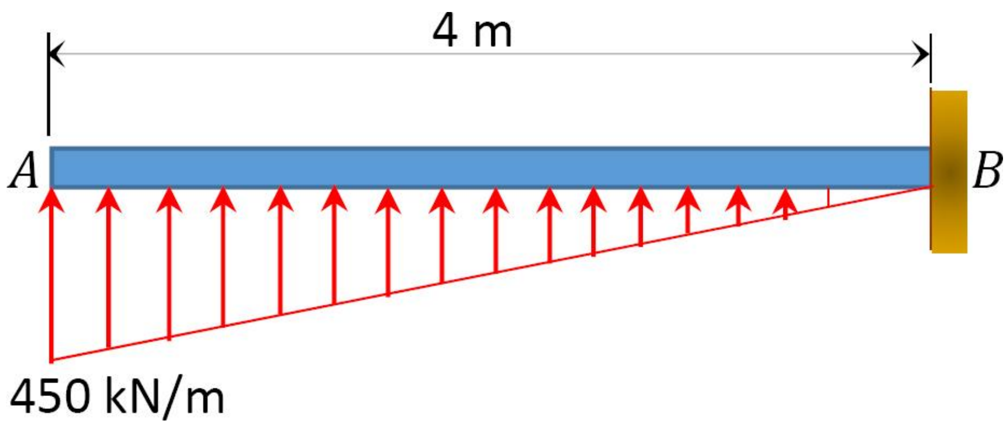


Fig. P3.3. Beam.

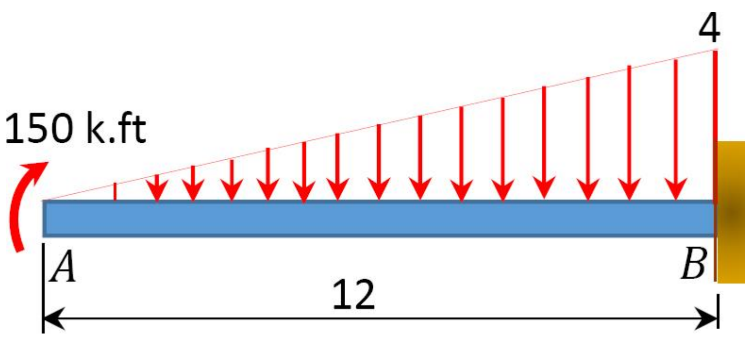


Fig. P3.4. Beam.

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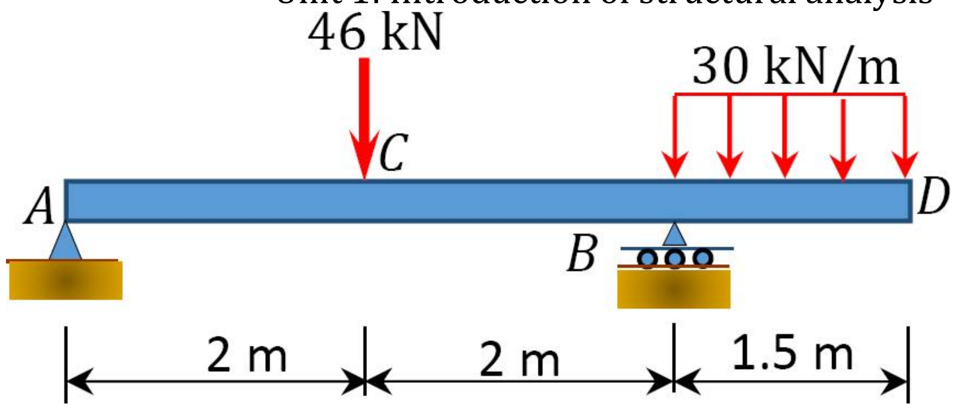

Fig. P3.5. Beam.

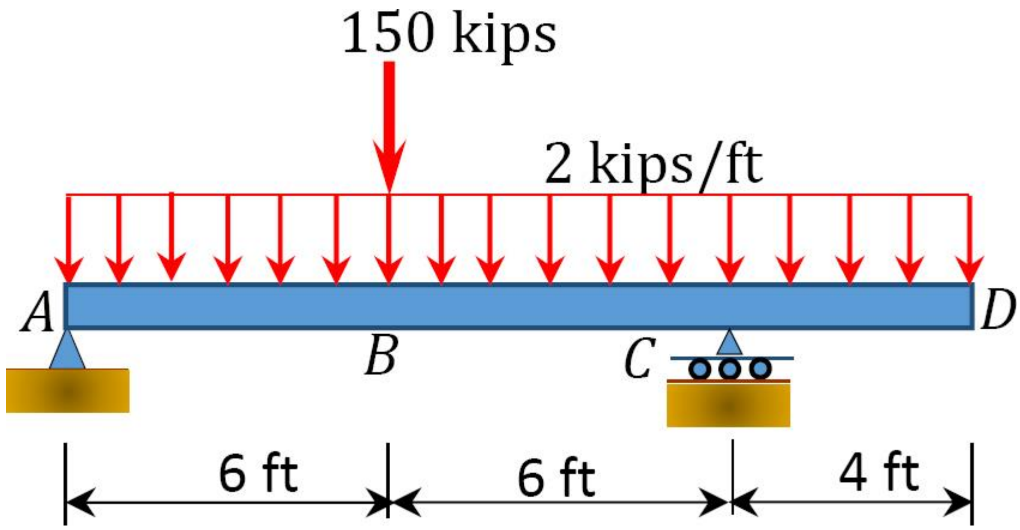

Fig. P3.6. Beam.

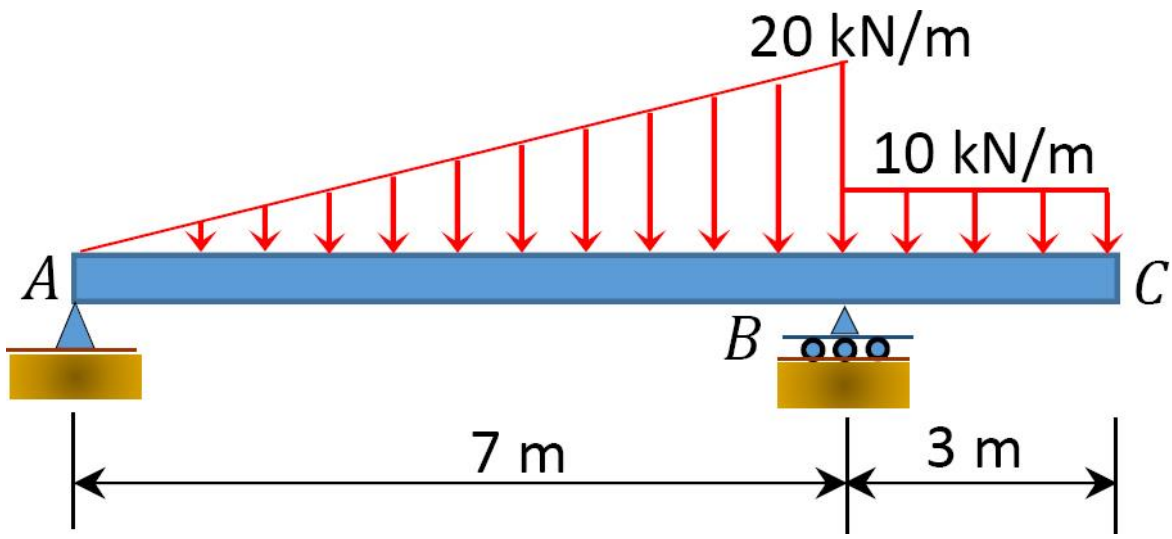

Fig. P3.7. Beam.

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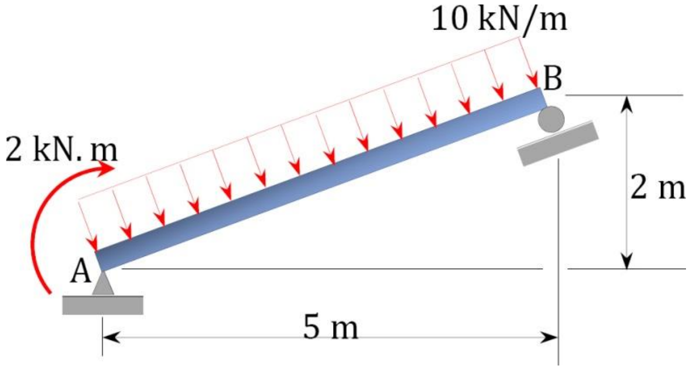

Fig. P3.8. Beam.

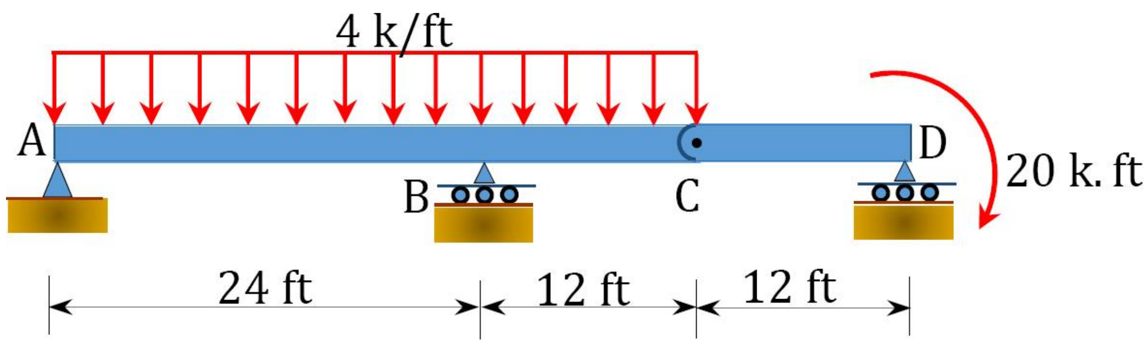

Fig. P3.9. Beam.

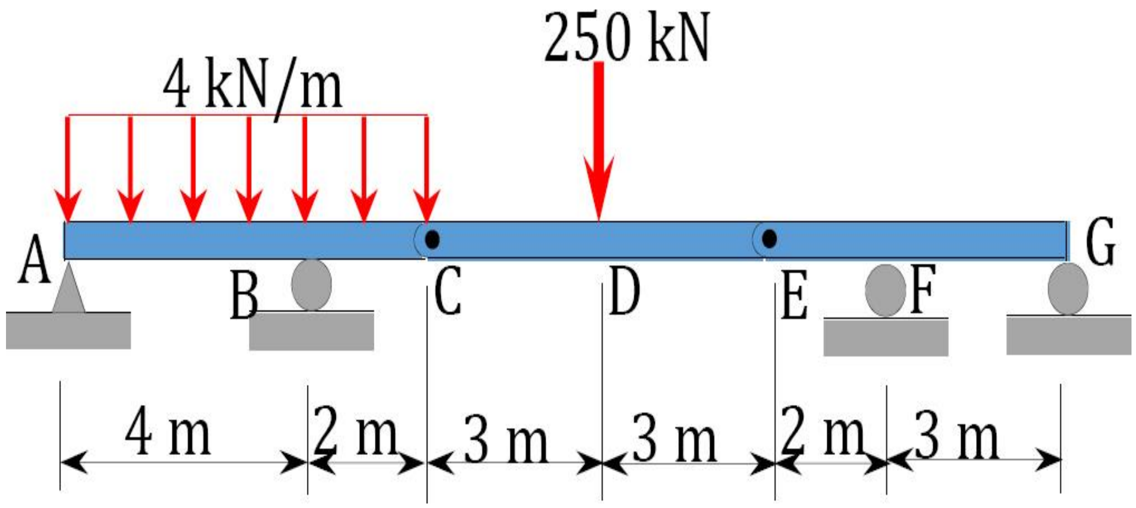

Fig. P3.10. Beam.

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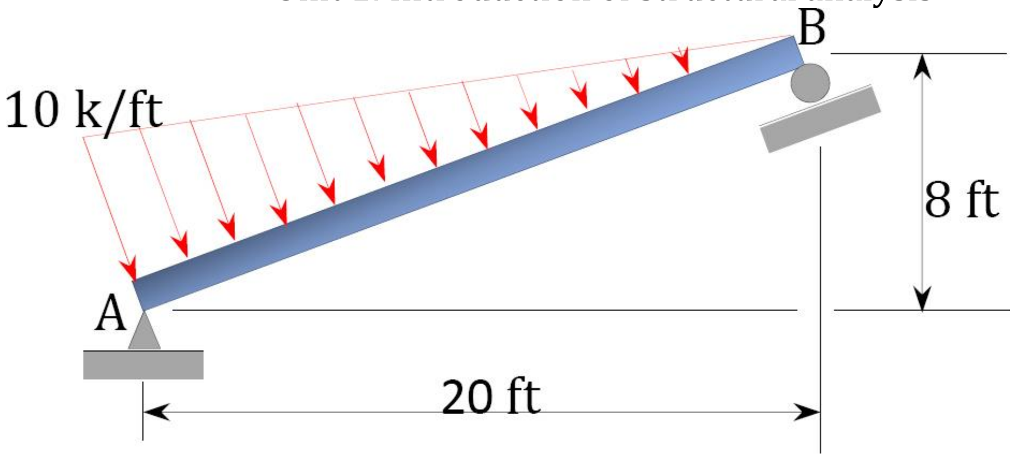

Fig. P3.11. Beam.

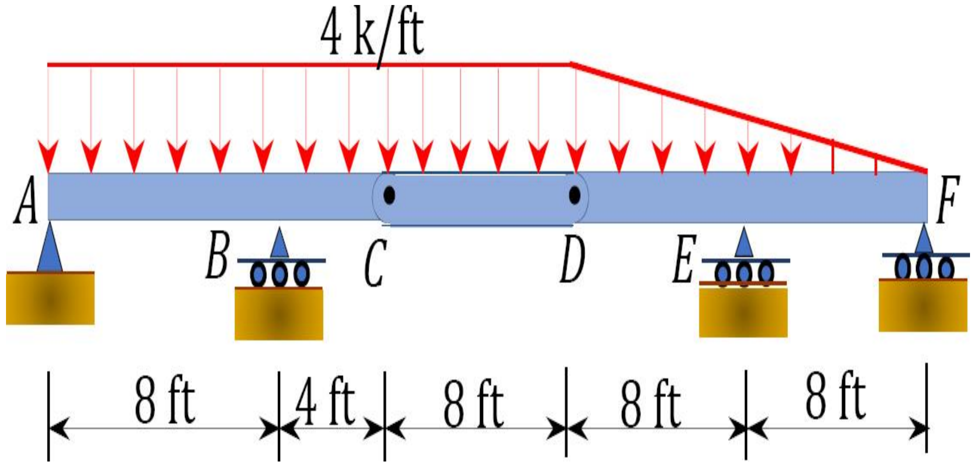

Fig. P3.12. Beam.

3.3. Determine the support reactions for the frames shown in Figure P3.13 through Figure P3.20.

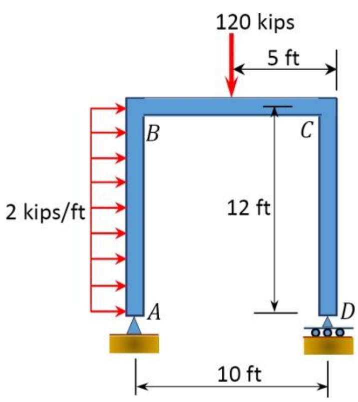

Fig. P3.13. Frame.

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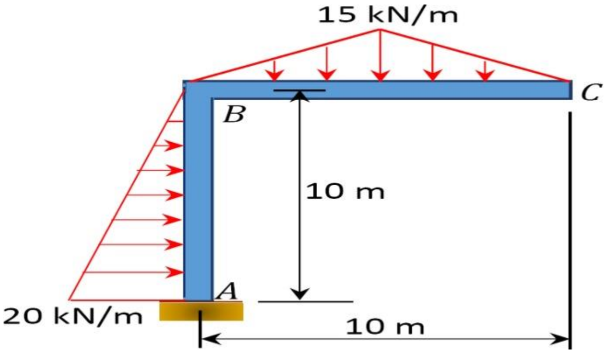

Fig. P3.14. Frame.

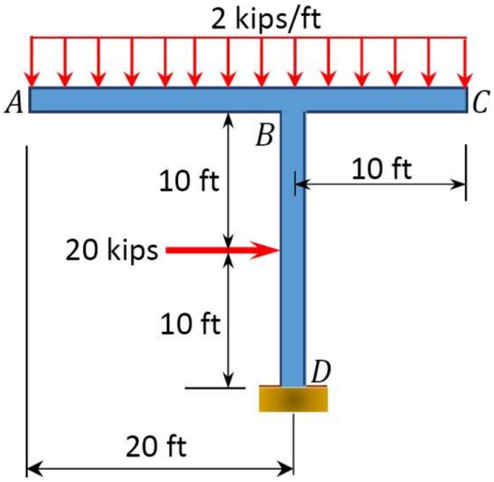

Fig. P3.15. Frame.

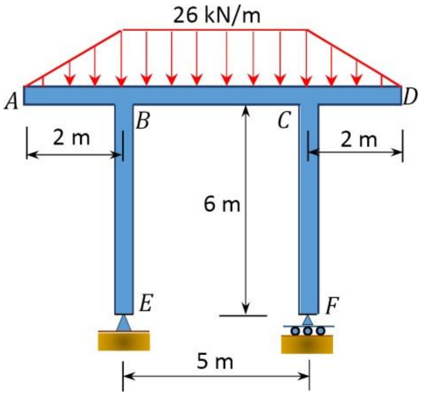

Fig. P3.16. Frame.

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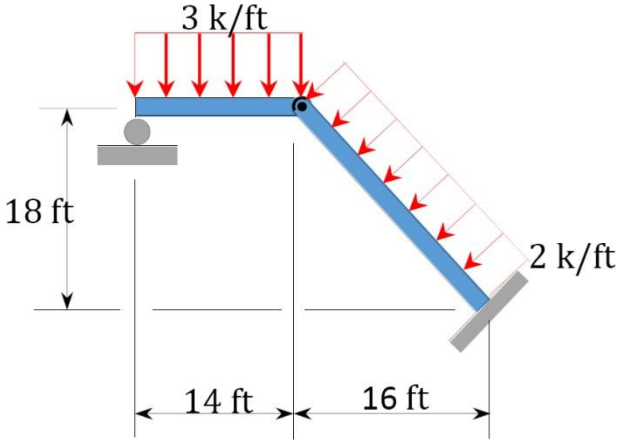

Fig. 3.17. Frame.

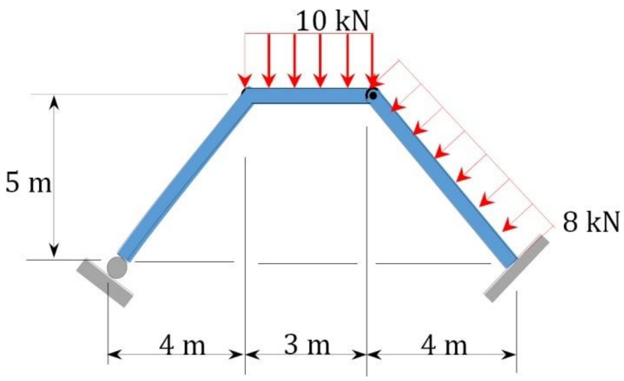

Fig. 3.18. Frame.

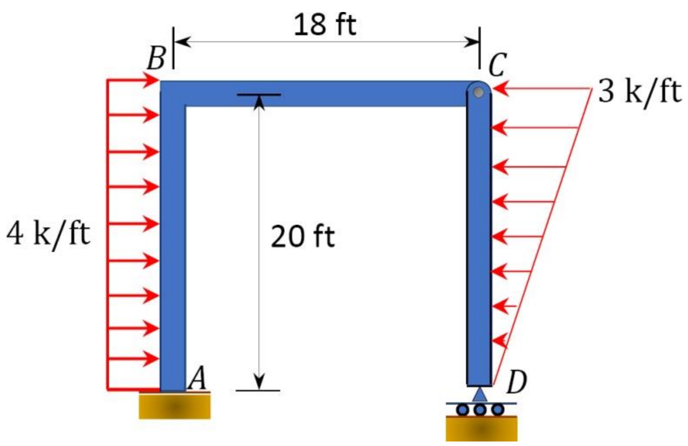

Fig. 3.19. Frame.

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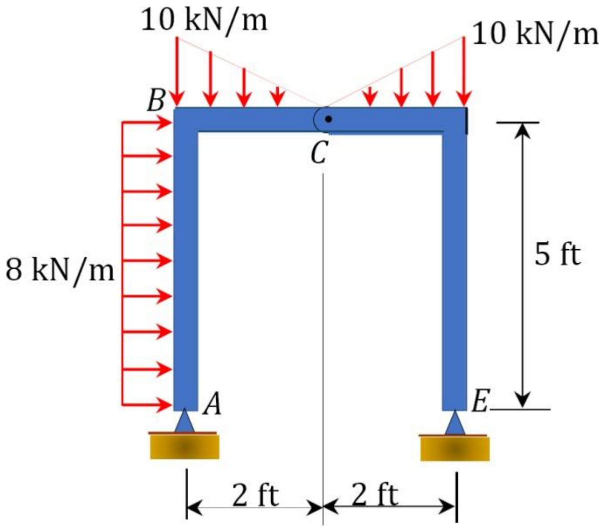

Fig. 3.20. Frame.

3.4 Determine the support reactions for the trusses shown in Figure P3.21 through Figure P3.27.

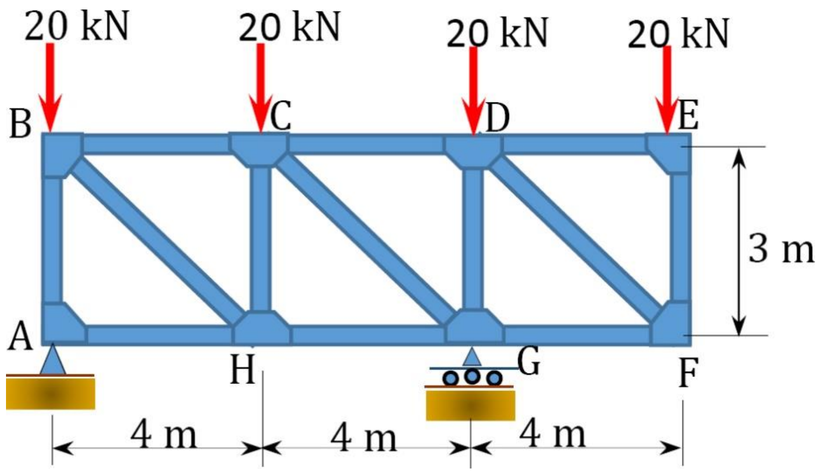

Fig. P3.21. Truss.

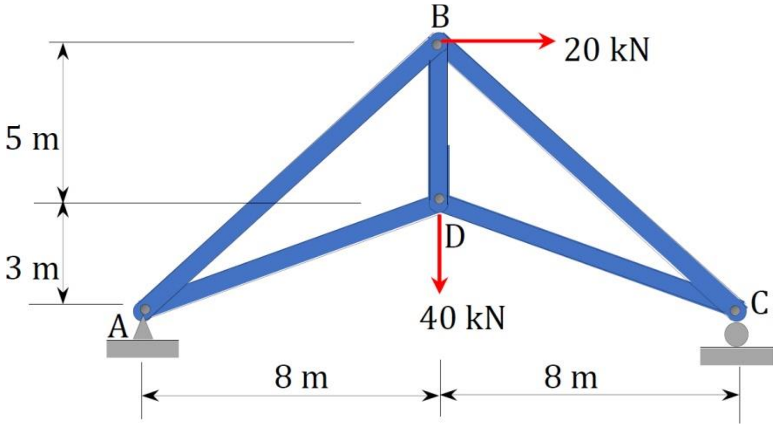

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Fig. P3.22. Truss.

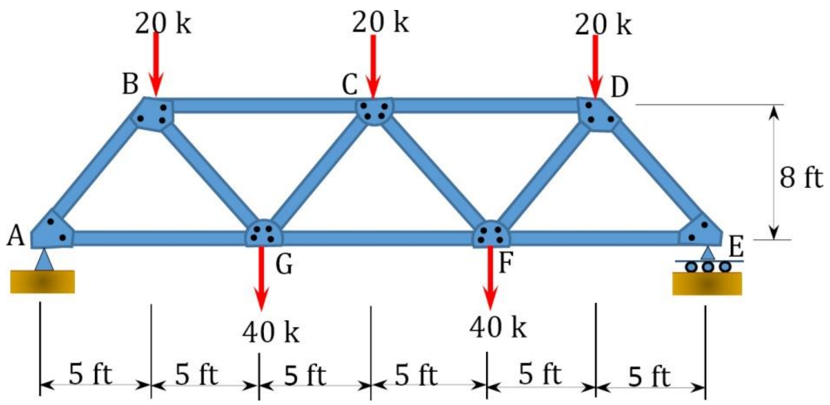

Fig. P3.23. Truss.

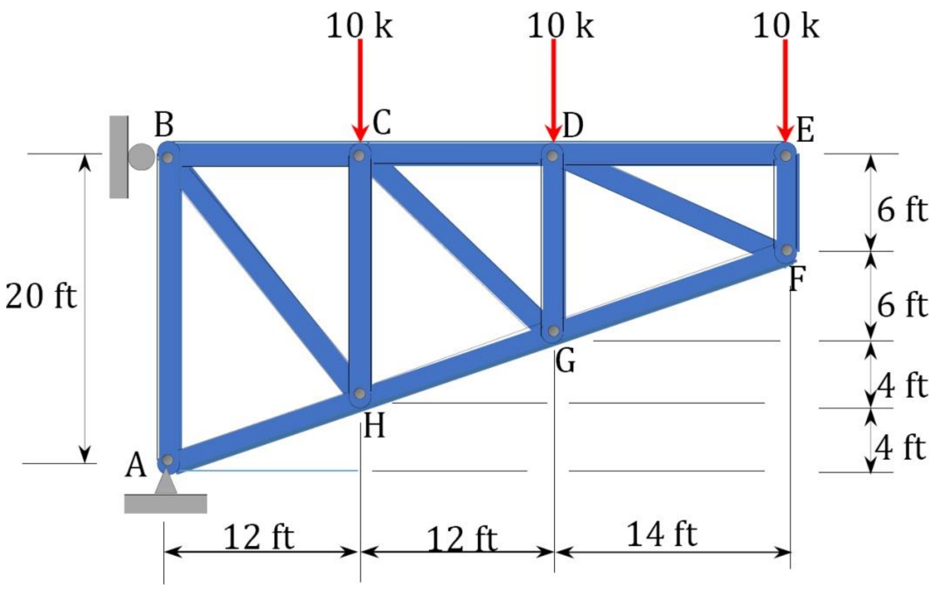

Fig. P3.24. Truss.

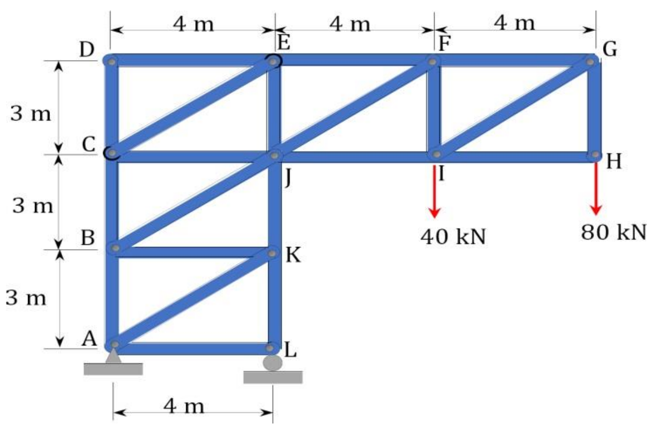

Fig. P3.25. Truss.

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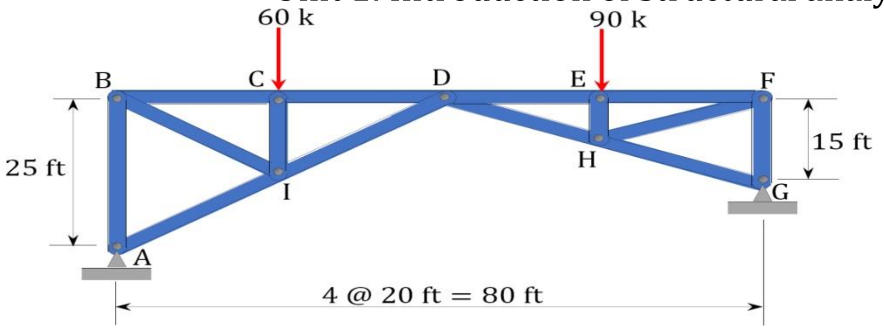

Fig. P3.26. Truss.

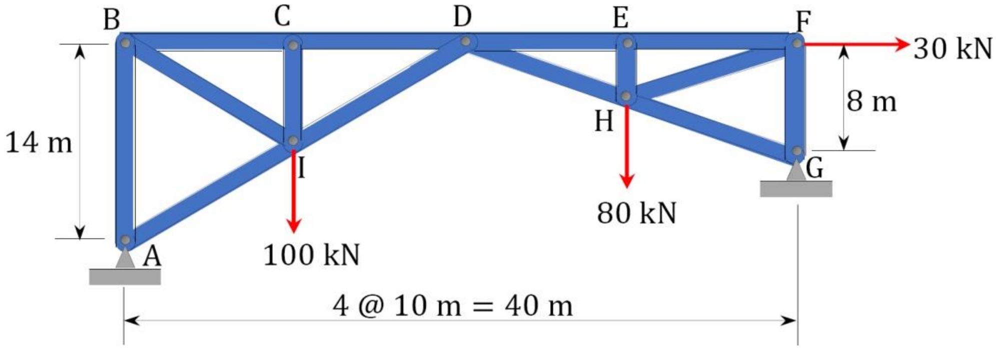

Fig. P3.27. Truss.

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