Syllabus

Positive real function; definition and properties, Properties of LC, RC and RL driving point functions, Synthesis of LC, RC and RL driving point immittance functions using Foster and Cauer first and second forms. Filters -Image parameters: Image impedance, characteristics impedance, image transfer parameter, Passive and active filter fundamentals, Low pass filters, High pass (constant K type) filters, Introduction to active filters.

Positive Real Function:

The significance of positive real functions is that if the driving point immitance (i.e. admittance or impedance) is a positive real function then only it is physically realizable using passive R, L and C components. Hence immitance function must be checked for positive realness before synthesizing.

For a function to be positive real function it has to satisfy the following basic properties,

- The given function F(s) is real for real s.
- The real part of F(s) is greater than or equal to zero, when the real part of s is greater than or equal to zero.

$$Re[F(s)] \geq 0 \quad \text{for} \quad Re[s] \geq 0$$

- The function F(s) is rational function.

The positive real function is also called Brune function. In addition to the basic properties, the positive real function has some more properties.

Properties of Positive Real Function:

Let

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}$$

The function F(s) is positive real function having following properties,

1. The coefficients of the numerator and denominator polynomials N(s) and D(s) in F(s) are real and positive.

Hence,

- F(s) is real when s is real.
- The complex poles and zeros of F(s) occur in complex conjugate pairs.
- The scale factor, N = N = a_0/b_0 is real and positive.

2. The poles and zeros of F(s) are having negative or zero real parts.

3. The poles of F(s) on the imaginary (jω) axis must be simple. Their residues must be real and positive. The same statement is true for the poles of 1/F(s).

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4. The degrees of the numerator and denominator polynomials in F(s) differ at the most by 1. So degrees m and n must differ at the most by 1. Thus the number of finite poles and finite zeros of F(s) differ at the most by 1.

5. The terms of lowest degree in the numerator and denominator polynomials of F(s) differ in degree at most by 1. So F(s) has neither multiple poles nor zeros at the origin.

6. If F(s) is positive real function then 1/F(s) is also positive real function. Thus for a network if driving point impedance function Z(s) is positive real then the driving point admittance function Y(s) = 1/Z(s) is also positive real.

7. The sum of positive real functions is also positive real. If two impedances are in series, the sum of the impedances is positive real. Similarly if two admittances are in parallel, their addition gives positive real admittance. Note that the difference in two positive real functions is not necessarily a positive real function.

These properties are nothing but necessary conditions for the given function to be positive real but are not sufficient conditions.

Procedure for Testing a Function for Positive Realness:

Let F(s) be the function to be tested for the positive realness, which is a ratio of two polynomials N(s) and D(s). Remove all the common factors in the numerator and the denominator before testing for the positive realness.

The testing procedure can be divided as,

1. Testing for necessary conditions
2. Testing for necessary and sufficient conditions

Inspection Test for Necessary Conditions:

By inspecting the given function, the following requirements are tested,

1. All the coefficients of different polynomials must be real and positive.
2. The degrees of numerator and denominator polynomials differ at most by 1.
3. Lowest degree in numerator and denominator differ at most by 1.
4. The imaginary axis poles and zeros of F(s) must be simple in nature. No multiple pole or zero should lie on the imaginary axis.
5. There should be no missing terms in numerator and denominator unless all even or all odd terms are missing.
6. The poles and zeros of F(s) must be located in the left half of s-plane.
7. There should not be multiple poles or zeros either at origin ($s = 0$) and/or at infinity ($s = \infty$).
8. The simple poles on $j\omega$ axis should have real and positive residues.

Test for Necessary and Sufficient Conditions:

The tests for necessary and sufficient conditions are,

1. The function F(s) must be real when s is real.

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2. If F(s) = N(s)/D(s)

then N(s) + D(s) must be Hurwitz.

For N(s) + D(s) to be Hurwitz, all the quotients obtained by expressing it in continued fraction expansion must be positive. Also the continued fraction expansion should not terminate abruptly.

3. Re [F(jω)] ≥ 0 for all ω.

The real part of F (jω) must be greater than equal to zero for all ω.

To test this, separate the numerator and denominator polynomials into even and odd parts.

$$F(s) = \frac{N(s)}{D(s)} = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)} = \frac{m_1 + n_1}{m_2 + n_2}$$

Multiply numerator and denominator by $m_2 - n_2$

$$F(s) = \frac{m_1 + n_1}{m_2 + n_2} \times \frac{m_2 - n_2}{m_2 - n_2}$$

$$= \frac{(m_1 m_2 - n_1 n_2) + (m_2 n_1 - m_1 n_2)}{m_2^2 - n_2^2}$$

$$= \frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2} + \frac{m_2 n_1 - m_1 n_2}{m_2^2 - n_2^2}$$

It is known that,

- Product of two even functions is an even function.
- Product of two odd functions is an even function.
- Product of an odd function and an even function is an odd function.

So as m_1 and m_2 are even, $m_1 m_2$ is even.
And n_1 and n_2 are odd, $n_1 n_2$ is even.
Also $m_1 n_2$ and $m_2 n_1$ is odd as the product of even and odd function.
Finally m^2_2 and n^2_2 are both even.
Hence in the above equation of F(s), first part is even while the second part is odd.

$$\text{Even part of } F(s) = \frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2}$$

$$\text{Odd part of } F(s) = \frac{m_2 n_1 - m_1 n_2}{m_2^2 - n_2^2}$$

Hence substituting s =jω in the even part gives the real part of F(jω) while substituting s =jω in odd part gives imaginary part of F(s).

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Even part of $F(s)|_{s=j\omega} = \text{Re}\,[F(j\omega)]$

We have to test $\text{Re}\,[F(j\omega)] \geq 0$

Now for $s = j\omega$, $m_2^2 - n_2^2|_{s=j\omega}$ is always positive.

Hence for $\text{Re}\,[F(j\omega)] \geq 0$ it is necessary that

$m_1 m_2 - n_1 n_2|_{s=j\omega} \geq 0$ for all $\omega \geq 0$

So $A(\omega)^2 = m_1 m_2 - n_1 n_2|_{s=j\omega} \geq 0$ for all $\omega \geq 0$

In most of the cases, the condition can be verified by factorising A (ω^2). If factorization is not sufficient for the required conclusion then A (ω^2) is plotted over sufficiently large range of ω and it is ensured that it is not negative.

Thus the testing procedure for positive realness of a function can be summarized as,

1. Check all the necessary conditions by inspection.

2. For all the poles and zeros of F(s) to be in the left half of s-plane or on the imaginary axis, N(s) and D(s) polynomials must be Hurwitz.

3. If F(s) has poles on imaginary axis, the residues at the poles must be real and positive.

Find the partial fractions of F(s) where $s = \pm j\omega_0$ is the pair of poles on imaginary axis.

$$F(s) = \ldots\ldots\ldots + \frac{k}{s - j\omega_0} + \frac{k^*}{s + j\omega_0}$$

So coefficients k and k˙ which are complex conjugates of each other must be real and positive.

4. Finally test that

$$A(\omega^2) = m_1 m_2 - n_1 n_2|_{s=j\omega} \geq 0 \quad \text{for all } \omega \geq 0$$

This can be done by factorising A (ω^2). This also can be tested by plotting A (ω^2) graph against ω^2 or using Sturm's theorem.

Driving Point Immittance Function:

The immitance function must be positive real function so that its synthesis can be done to obtain an electrical network, using passive elements. We have discussed the tests to confirm the positive realness of a given function. Let us study now how to synthesize the given Driving Point Immittance Function (impedance or admittance) which is positive real.

There are three types of passive elements which are an inductor (L), a capacitor (C) and a resistor (R). A given function can be synthesized using any two types of passive elements. Thus synthesized network can be LC network, RC network or RL network.

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A network using any two types of passive elements can be synthesized generally in two forms called,

1. **Foster form and**
2. **Cauer form.**

These forms are used for the network realization because the network is realized using minimum number of passive elements using these basic forms. Hence these forms are called **canonical** or **simple forms** of realization. In each of these forms there are two sub forms. The foster form is subdivided as Foster I form and Foster II forms while the Cauer form is subdivided as Cauer I and Cauer II forms.

The following table is useful while realizing the network by any of the four forms mentioned above

Element	Z(s)	Y(s)
Resistance R	R	$\frac{1}{R}$ = G conductance
Inductance L	sL	$\frac{1}{sL}$
Capacitance C	$\frac{1}{sC}$	sC

Table 7.1

Foster I Form:

The Foster I form uses the partial fractions of the driving point impedance function Z(s). The partial fraction expansion gives the equation of Z(s) as summation of the various impedance functions.

$$Z(s) = Z_1(s) + Z_2(s) + \ldots\ldots + Z_n(s)$$

Thus a network can be realized by connecting the impedances Z_1,Z_2, ……..Z_n in series.

Each impedance individually can be a series combination or parallel combination of the elements, which is to be identified. The identification of various elements existing in the different impedance function can be done by referring the Table 7.1. While finding the partial fractions, make sure that the degree of numerator is less than the degree of denominator. In this form, number of times, we get the functions after partial fraction expansion as,

$$Z'(s) = \frac{A}{s+B}$$

This can be synthesized by expressing it as

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$$Z'(s) = \frac{1}{\frac{s}{A} + \frac{B}{A}}$$

We know

$$Z'(s) = \frac{1}{Y'(s)}$$

$$Y'(s) = \frac{s}{A} + \frac{B}{A} = Y_1(s) + Y_2(s)$$

The addition of two admittances indicate that there are two branches in parallel.

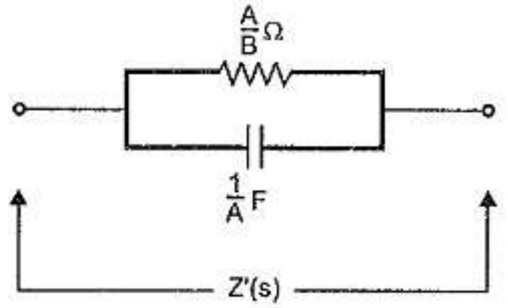

Fig. 7.3

Referring to the Table 7.1, $Y_1(s) = s/A$ represents a capacitor of 1/A F while $Y_2(s) = B/A$ represents conductance of B/A mho i.e. a resistance of A/B ohms. Hence Z'(s) can be realized as parallel combination of R and C as shown in the Fig. 7.3.
Note that this example illustrates how to synthesize a particular function obtained in the partial fraction form. It does not mean that every time Foster I form gives RC networks.

Foster II Form:

The Foster II form uses the partial fractions of the driving point admittance function Y(s) = 1/Z(s). The partial fraction expansion gives the equation of Y(s) as summation of the various admittance functions.

$$Y(s) = Y_1(s) + Y_2(s) + ... + Y_n(s)$$

Thus a network can be realized by connecting the admittances Y_1, Y_2, …… Y_n in parallel.
Each admittance individually can be a series combination or parallel combination of the various elements. These combinations are to identified to realize the network. This can be easily done by referring the Table 7.1, as illustrated above.

Cauer I Form:

Basically Cauer form uses continued fraction expansion of the Driving Point Immittance Function. The Cauer I uses the continued fraction expansion of the driving point impedance function Z(s). In this form, the numerator and the denominator are arranged in the descending powers of s, starting from highest to lowest power of s.

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This form gives the **ladder structure** of the network with alternate series and shunt arms. Hence this form of realization is also called **ladder realization**.
The continued fraction expansion gives the form as,

$$Z(s) = \underset{\text{(series)}}{Z_1(s)} + \cfrac{1}{\underset{\text{(shunt)}}{Y_2(s)} + \cfrac{1}{Z_3(s) + \ldots\ldots}}$$

The continued fraction expansion can be obtained by the division and inversion procedure, used earlier for testing the Hurwitz polynomials.

In the continued fraction form of Z(s), the quotient of first division gives an impedance which is a series arm and then the quotients give alternately Y(s) and Z(s) terms representing shunt and series arms respectively.

The elements of each Z(s) and Y(s) obtained can be identified using the Table 7.1.

Remember that if the degree of numerator is same or less than the denominator, there is possibility of getting negative coefficients in the continued fraction expansion. In such case restart with the inversion and first quotient in such case represents admittance Y(s) which is a shunt arm. The series arm is absent in such case.

The Cauer I form is low pass structure of the network.

If the driving point admittance function Y(s) is expressed in continued fraction expansion with division at start, first quotient represents admittance Y(s) in shunt arm and then, alternately impedances and admittances in series and shunt arms respectively. Starting with inversion in such case gives impedance Z(s) in series arm as the first quotient, and then alternately admittances and impedances in shunt and series arms.

Cauer-II Form:

The Cauer II form also uses continued fraction expansion of the driving point immittance function either Z(s) or Y(s). In this form, the numerator and the denominator are arranged in the ascending powers of s, starting from lowest to highest power of s.

The continued fraction expansion gives the similar form as,

$$Z(s) = \underset{\text{(series)}}{Z_1(s)} + \cfrac{1}{\underset{\text{(shunt)}}{Y_2(s)} + \cfrac{1}{Z_3(s) + \ldots\ldots}}$$

In this form also, there is a possibility of getting negative coefficients in the expansion procedure. In such a case, restart with the inversion, which gives Y(s) as the first quotient which indicates shunt arm, without a series arm in the circuit. Similarly driving point admittance function Y(s) also can be synthesized using Cauer II form.
The Cauer II form is basically high pass structure.

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Remember that if Y(s) is to be synthesized and expansion starts with the division, first quotient gives Y(s) in shunt arm and if starts with the inversion, first quotient gives Z(s) in series arm and vice versa for Z(s).

LC Immittance Function:

The LC Immittance Function can be LC impedance functions denoted as $Z_{LC}(s)$ or LC admittance functions denoted as $Y_{LC}(s)$.
A LC network does not contain power dissipative components i.e. resistances and only consists of reactive elements L and C components. Hence such network is also called a **reactance network** or **lossless network**.
Consider a driving point impedance function of LC one port network represented as the ratio of two polynomials in s.

$$Z_{LC}(s) = \frac{N(s)}{D(s)} = \frac{m_1(s) + n_1(s)}{m_2(s) + n_2(s)}$$

Where m_1, n_1 are even and odd parts of N(s) while m_2, n_2 are the even and odd parts of D(s) respectively.
For a purely reactive element jωL or 1/jωC, the real part is zero.

$$\text{Re}\,[Z_{LC}(j\omega)] = 0$$

We have seen that the real part of F(jω) is its even part given by,

$$\text{Re}\,[Z_{LC}(j\omega)] = \text{Even}\,[Z_{LC}(j\omega)] = \frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2}$$

But real part must be zero for LC network.

$$\therefore \quad m_1 m_2 - n_1 n_2 = 0$$

Thus to satisfy this equation we must have,

$$1.\ m_1 = n_2 = 0 \quad \text{i.e.} \quad Z_{LC}(s) = \frac{n_1}{m_2}$$

$$2.\ m_2 = n_1 = 0 \quad \text{i.e.} \quad Z_{LC}(s) = \frac{m_1}{n_2}$$

So it can be concluded that the driving point impedance function of LC network is the ratio of odd to even polynomial or even to odd polynomial. This is a very important property of LC network.

For example,

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For example, $Z_{LC}(s) = \frac{s^4 + 10s^2 + 9}{s^3 + 4s} = \frac{\text{Even}}{\text{Odd}}$

$$\therefore Z_{LC}(s) = \frac{(s^2+1)(s^2+9)}{s(s^2+4)}$$

So poles at $s = 0, \pm j\,2$

Zeros at $s = \pm j, \pm j\,3$

So poles at s = 0, ± j 2

Zeros at s = ± j, ± j 3

The pole zero plot is shown in the Fig. 7.4.

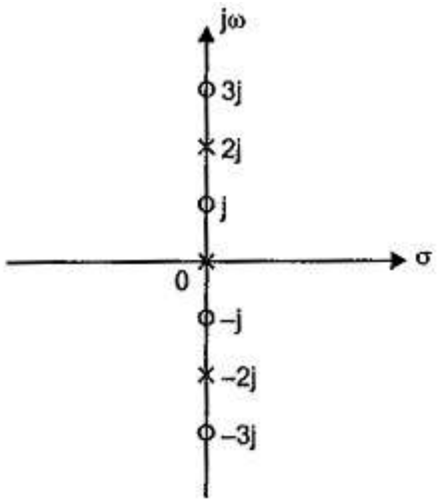

Fig. 7.4

From this impedance function of LC network, let us list the various other properties of LC networks.

Properties of LC Immittance Function:

Referring to the pole zero plot shown in the Fig. 7.4, the properties of $Z_{LC}(s)$ and $Y_{LC}(s)$ functions can be stated as,

1. The LC Immittance Function is always a ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple. There are no multiple poles or zeros either at origin or infinity or at any point.
3. The poles and zeros are located on the jω axis only.
4. The poles and zeros interlace (alternate) each other on the jω axis. There are no consecutive poles or zeros on the jω axis.
5. The imaginary poles and zeros occur in the form of complex conjugate pairs.
6. The highest powers of numerator and denominator must differ by unity.
7. The lowest powers of numerator and denominator must differ by unity.

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8. There must be either a pole or zero at the origin and infinity. As the function is the ratio of even to odd polynomials, if the highest power of numerator is 2m then of denominator is 2m —1 which gives pole at ∞ or it can be 2m +1 which gives zero at ∞. And as lowest powers also differ by unity, there is pole or zero at the origin.
9. Residues at the imaginary axis poles are real and positive.
10. The number of elements required in any of the four forms of realization is equal to the highest power of s in the LC Immittance Function as a whole.
11. The slope of the graph of reactance against frequency is always positive.

We know that there is either pole or zero at the origin.

$$Z(s) = \frac{(s^2+1)(s^2+9)}{s(s^2+4)}$$

Consider

So there is pole at the origin. Let us obtain the graph of X(ω). At the starting pole at s= 0 i.e. ω = 0 the X(ω) is infinity. As ω increases, X(ω) also increases and at next critical frequency $\omega = \omega_2$ where there is a zero, X(ω) becomes zero. As ω further increases from $\omega = \omega_2$, X(ω) increases and becomes infinity at $\omega = \omega_3$ where there is a pole. At this frequency, X(ω) suddenly changes the sign and goes from + ∞ to — ∞, such that as we pass through $\omega = \omega_3$, slope of the graph always remains positive. It becomes zero at $\omega = \omega_4$ where there is a zero. The nature continues such that the slope d/dω [X(ω)] always positive. The nature is shown in the Fig. 7.5 for the Z(s) considered.

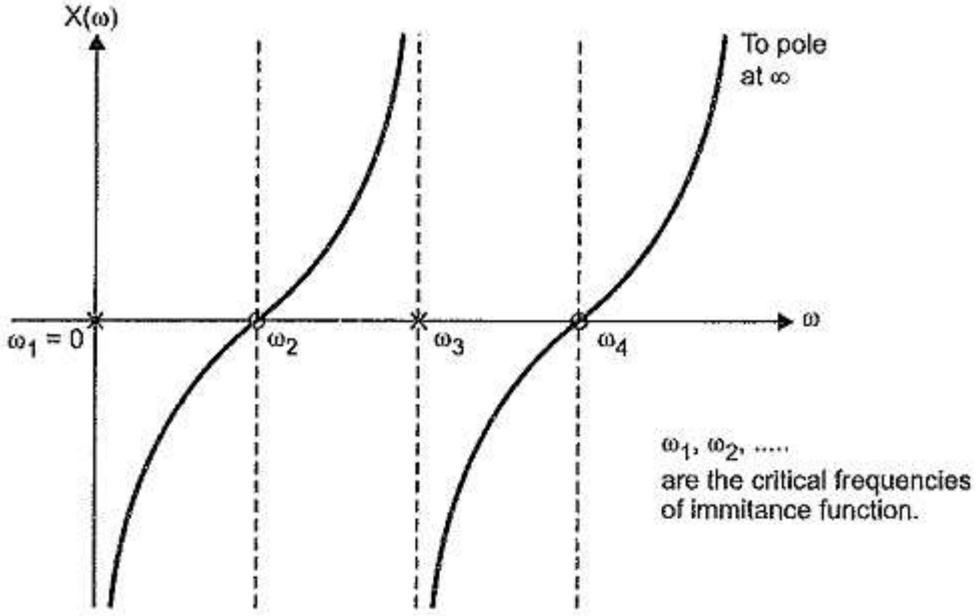

Fig. 7.5

Between ω_1 and ω_2, nature of X(ω) is capacitive while between ω_2 and ω_3 it is inductive. At $\omega_3 = 2$, it changes from inductive to capacitive suddenly so that slope of the graph remains always positive.

For a zero at the origin, the reactance curve takes the form as shown in the Fig. 7.6.

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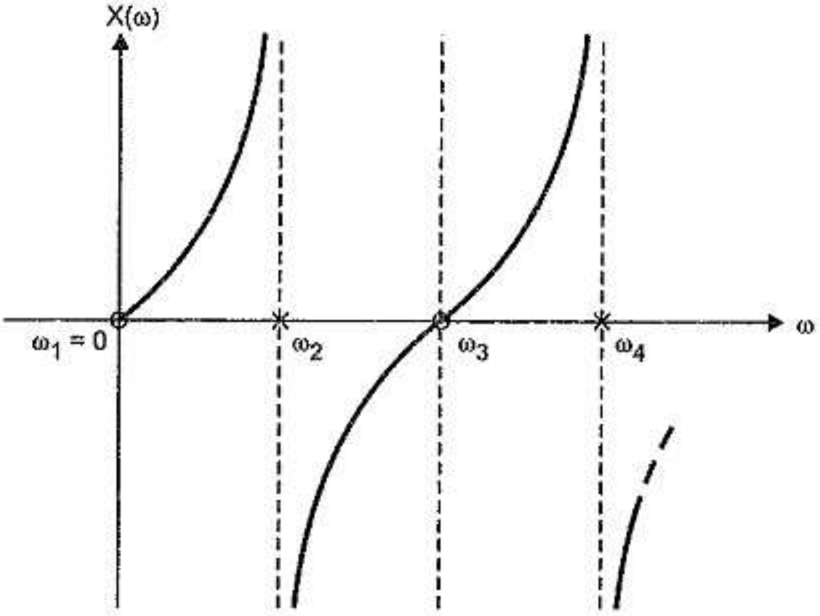

Fig. 7.6

For a LC function, $$\frac{d[X(\omega)]}{\omega} > 0 \text{ for } -\infty < \omega < \infty$$

From the nature of the reactance curves shown in the Fig. 7.5 and 7.6 with pole at origin and zero at origin respectively, it can be seen that curve has to change its sign at the poles to maintain the positive slope.

Hence to maintain the positive slope, the poles and zeros must separate each other and in such a way that the poles and the zeros alternate along the real frequency axis. This is called **Separation Property** or **Foster Reactance Theorem** for the reactance functions.

Thus if $$Z_{LC}(s) = \frac{H\left(s^2+\omega_{z_1}^2\right)\left(s^2+\omega_{z_2}^2\right)\ldots\ldots}{s\left(s^2+\omega_{p_1}^2\right)\left(s^2+\omega_{p_2}^2\right)\ldots\ldots}$$

then H is scale factor. Now ω_{z1} can be zero which gives zero at the origin or has finite existence if there is pole at the origin.

According to the **Reactance theorem** pole and zero frequencies must satisfy the relation given by,

$$0 < \omega_{z_1} < \omega_{p_1} < \omega_{z_2} < \omega_{p_2} < \ldots\ldots$$

Realization of Immitance Function of LC Networks:

The realization of driving point immitance functions of LC networks can be done by any of the four forms discussed earlier. The forms which are used are,

1. **Foster I form**
2. **Foster II form**
3. **Cauer I form**
4. **Cauer II form**

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RC Driving Point Impedance function:

As the name indicates, the RC networks consist of only R and C components. There is no inductor in RC networks. The RC Driving Point Impedance function is denoted as $Z_{RC}(s)$. The properties of RC Driving Point Impedance function and the properties of driving point admittance function of RL network are identical. Thus the properties $Z_{RC}(s)$ and $Y_{RL}(s)$ are same. There are no complex poles in RC network function. The poles and zeros alternate each other and are located in left half of s plane. To understand the properties of RC network function consider a driving point impedance function of RC network as,

$$Z_{RC}(s) = \frac{(s+1)(s+4)}{s(s+2)}$$

The poles are at s = 0, – 2

The zeros are at s = –1, – 4

The pole-zero plot is shown in the Fig. 7.11.

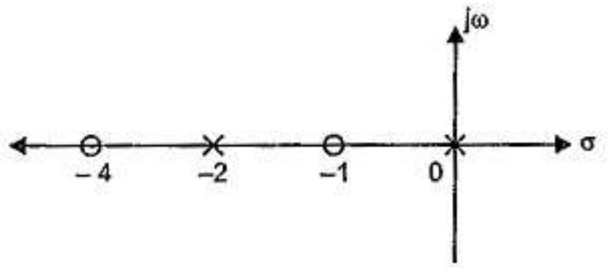

Fig. 7.11

Properties of RC Driving Point Impedance Function:

Referring to the pole zero plot of $Z_{RC}(s)$ function considered, the various properties of RC Driving Point Impedance function can be stated as,

1. The poles and zeros are simple. There are no multiple poles and zeros.
2. The poles and zeros are located on negative real axis.
3. The poles and zeros interlace (alternate) each other on the negative real axis.
4. We know that the poles and zeros are called critical frequencies of the The critical frequency nearest to the origin is always a **pole**. This may be located at the origin.
5. The critical frequency at a greatest distance away from the origin is always a zero, which may be located at ∞ also.
6. The partial fraction expansion of $Z_{RC}(s)$ gives the residues which are always real and positive.
7. There is no pole located at infinity.
8. The slope of the graph of $Z(\sigma)$ against σ is always negative.
9. There is no zero at the origin.
10. The value of $Z_{RC}(s)$ at s=0 is always greater than the value of $Z_{RC}(s)$ at s = ∞.

$$Z_{RC}(0) > Z_{RC}(\infty)$$

It can, be seen that for a Z_{RC} considered $Z_{RC}(0) = \infty$ while $Z_{RC}(\infty) = 1$.
Consider a simple RC network as shown in the Fig. 7.12.

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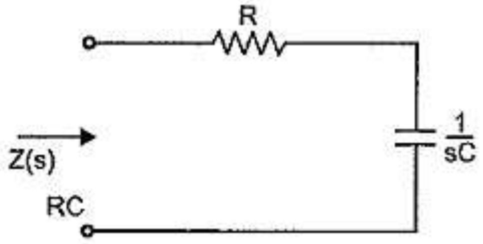

Fig. 7.12

$$Z_{RC}(s) = R + \frac{1}{sC} = \frac{1 + sRC}{sC}$$

Let us plot $Z_{RC}(\sigma)$ against σ where s = σ.

$$Z_{RC}(\sigma) = \frac{1 + \sigma RC}{\sigma C}$$

To find the slope of $Z_{RC}(\sigma)$ against σ find d $Z_{RC}(\sigma)$/dσ

$$\therefore \quad \frac{d\, Z_{RC}(\sigma)}{d\sigma} = \frac{(\sigma\, C\; RC) - (1 + \sigma\, RC)(C)}{(\sigma\, C)^2}$$

$$= \frac{\sigma\, C^2 R - C - \sigma\, C^2\, R}{\sigma^2\, C^2} = -\frac{C}{\sigma^2\, C^2}$$

$$\therefore \quad \frac{d\, Z_{RC}(\sigma)}{d\sigma} = -\frac{1}{\sigma^2\, C} < 0$$

Thus the slope of $Z_{RC}(\sigma)$ against σ is always negative for any value of σ.
The graph of Z(σ) against σ for the RC network function is shown in the Fig. 7.13.

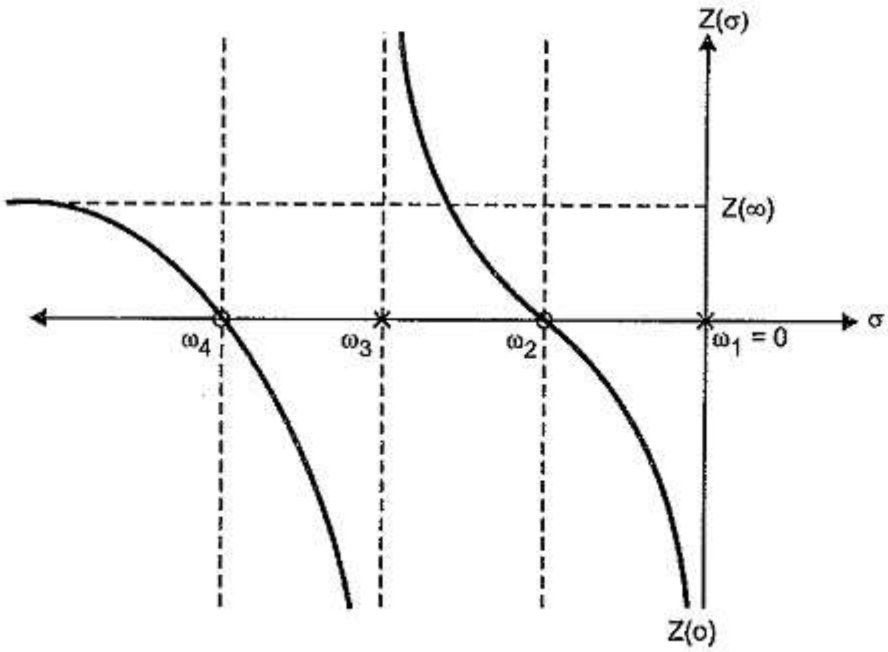

Fig. 7.13

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$\omega_1, \omega_2, \omega_3$ and ω_4 are the critical frequencies. At the critical frequencies like ω_3, the $Z(\sigma)$ changes its sign suddenly such that the slope always remains negative. The value of $Z(\infty)$ is constant so graph runs parallel to the σ axis, finally.

The nature of $Z(\sigma)$ against σ graph when there is no pole at the origin is shown in the Fig. 7.14.

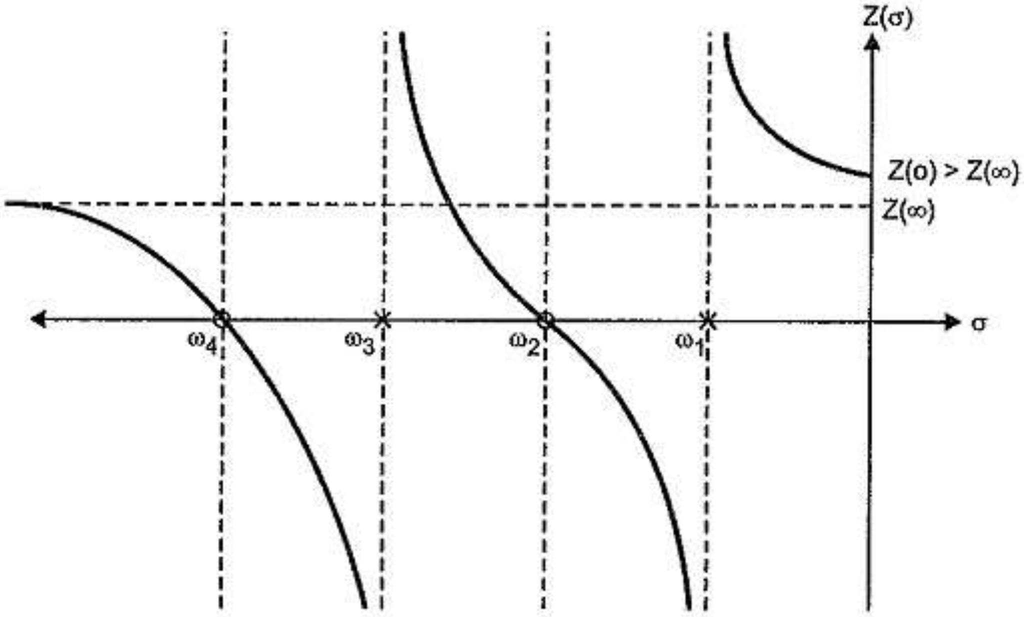

Fig. 7.14

All the properties of driving point admittance function of RL network [$Y_{RL}(s)$] are exactly identical to the properties of driving point impedance function of RC network [$Z_{RC}(s)$].

Realization of Impedance Function of RC Network:

As mentioned earlier, the realization of $Z_{RC}(s)$ function can be achieved using Foster I, Foster II, cauer I or cauer II form. Remember that the number of elements are not equal to highest power of s in overall Z(s) for RC networks.

RL Driving Point Impedance:

The RL networks consist of only R and L components. There is no capacitor present in such networks. The RL Driving Point Impedance of such networks is denoted as $Z_{RL}(s)$. The properties of driving point impedance function of RL networks [$Z_{RL}(s)$] and the driving point admittance function of RC networks [$Y_{RC}(s)$] are exactly identical.

The RL impedance function is dual of RC admittance function. There are no complex poles in RL network functions and poles and zeros are located in left half of s-plane.

Consider the RL Driving Point Impedance network as,

$$Z(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

The poles are at s = – 2 and – 4 while the zeros are at s = – 1 and – 3.

The pole-zero plot is as shown in the Fig. 7.19.

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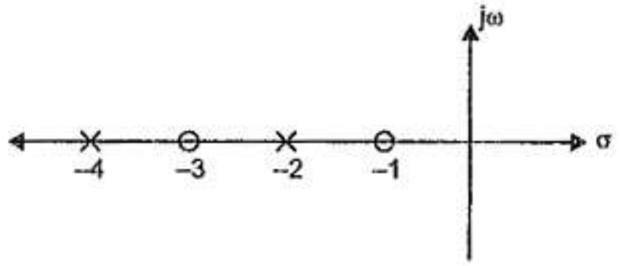

Fig. 7.19

Properties of RL Driving Point Impedance Functions:

Referring to the pole zero plot of $Z_{RL}(s)$ function considered above, the various properties of RL, driving point impedance functions can be stated as,

1. The poles and zeros are simple. There are no multiple poles and zeros.
2. The poles and zeros are located on negative real axis.
3. The poles and zeros interlace each other on the negative real axis.
4. The poles and zeros are the critical frequencies. The critical frequency nearest to the origin is always a zero, which may be located at the origin.
5. The critical frequency at a greatest distance away from the origin is always a pole, which may be located at infinity also.
6. Partial fraction expansion of $Z_{RL}(s)$ gives the residues which are negative and real hence to obtain positive residues the expansion of $Z_{RL}(s)/s$ is obtained.
7. There can not be a pole at the origin.
8. The slope of the graph of z (σ) against σ is always positive.
9. The value of $Z_{RL}(s)$ at s = 0 is always less than the value of $Z_{RL}(s)$at s = ∞.

$$Z_{RL}(0) < Z_{RL}(\infty)$$

For the example considered $Z_{RL}(0) = 3/8$ while $Z_{RL}(\infty) = 1$.

It can be easily verified from a simple RL network that the slope of the graph Z(σ) against σ is always positive.

$$\frac{dZ(\sigma)}{d\sigma} > 0$$

The graph of Z(σ) against σ for the RL network function without a zero at the origin is shown in the Fig. 7.20.

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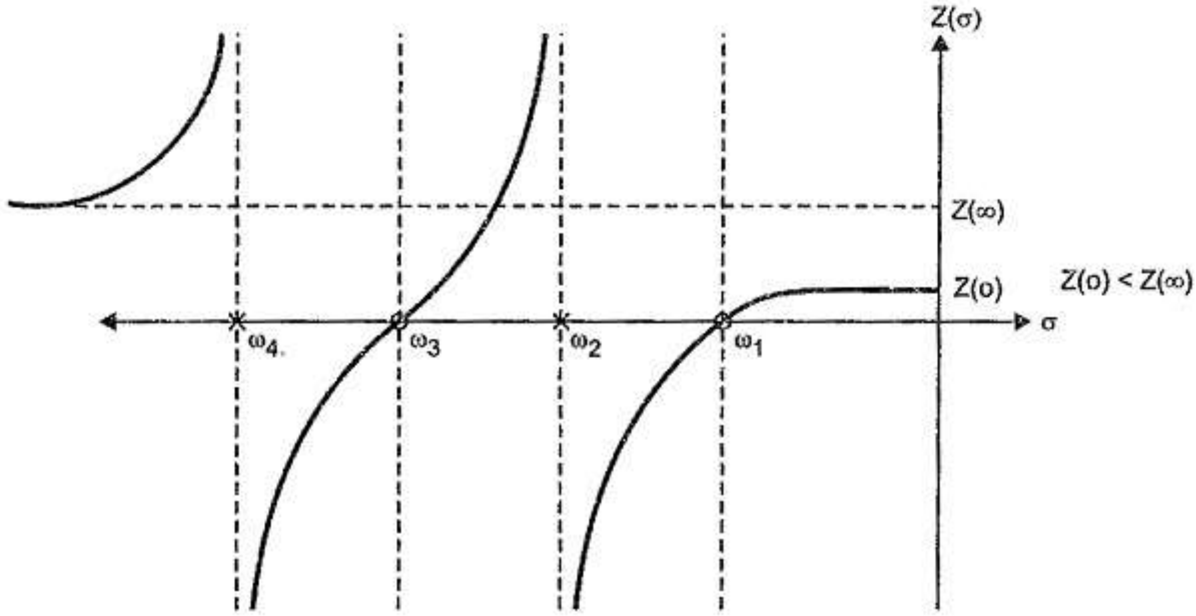

Fig. 7.20

$\omega_1, \omega_2, \omega_3$ and ω_4 are the critical frequencies. At the critical frequencies like ω_2 and ω_4 the sign of Z(σ) changes suddenly such that the slope always remains positive.

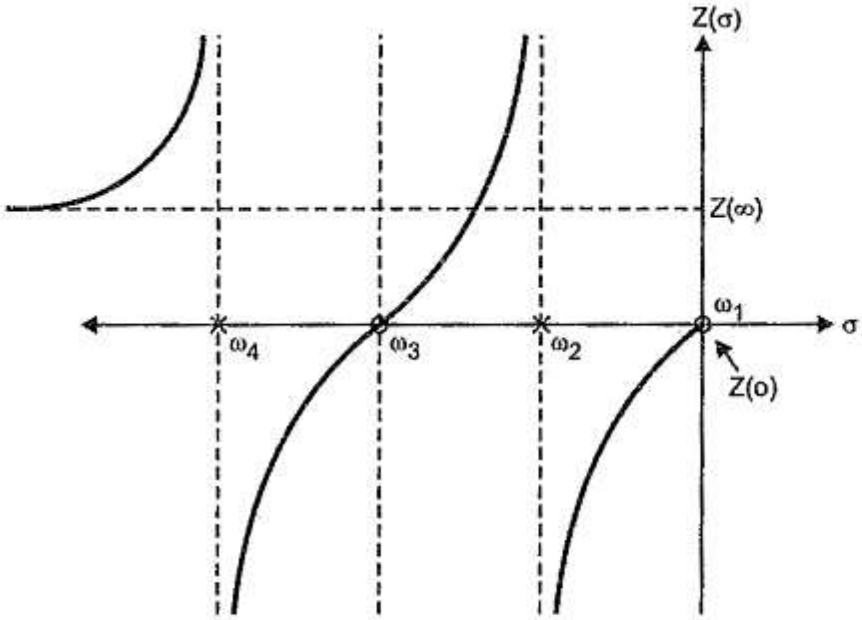

Fig. 7.21

The nature of Z(σ) against σ graph for the RL network function when there is a zero at the origin is shown in the Fig. 7.21.

Realization of Impedance Function of RL Network:

The realization of $Z_{RL}(s)$ function can be obtained using Foster I, Foster II, Cauer I and Cauer II forms. The number of elements are not equal to highest power of s in the overall Z(s) for RL networks.

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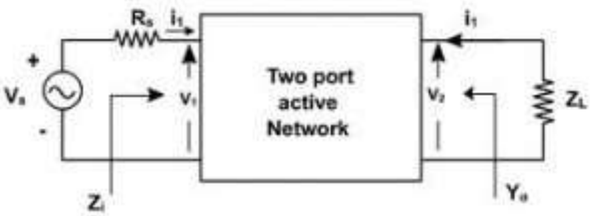
Image Impedance

It is the impedance which when connected to the input and the output of the transducer, it will make both the impedances equal at the input and the output terminal. It is basically the concept which is used in the field of the network analysis and design and also in the field of the filter design. It applies to the seen impedance which is determined by looking through the ports of the network.

The Two-port network can be properly used to describe the concept of the image impedance in the better way.



two port network

The impedance z_{i1} – when considered from the port 1

Z_{i2} –image impedance when considered from the port 2

The image impedance will not be equal until the network is the symmetrical network or anti-symmetrical with respect to the ports.

Characteristic impedance

The characteristics impedance also known as the surge impedance is usually considered in the case of the transmission line and is represented as Z_0. The characteristics impedance is defined as the ratio of the amplitude of the voltage and the current taking the consideration of the single wave through the line. The surge impedance is usually allocated through the transmission line with its geometry and the material. It is to be noted that this impedance is independent of the line length.SI unit – ohm

Image transfer coefficient

It is usually considered for the linear passive type of the two-port network, such network must be terminated with the image impedance of the network. Let

V_1 – voltage at the input terminal

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I_1 – current at the input terminal

V_2 – voltage at the output terminal

I_2 – current at the input terminal

Hence, the image transfer coefficient can be calculated as half the logarithm of the product of V_1 and I_1 divided by the product of the V_2 and I_2.

Represented as,

½ log ((V_1 I_1) / (V_2 I_2))

Filters

A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate the others. As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies. Filters are the circuits used to allow to us to select one desired signal out of a multitude of broadcast signals in the environment.

Filters are classified in two types:- • Active filters • Passive filters

A filter is a passive filter if it consist of only passive elements R, L and C.

It is said to be an active filter if it consists of active elements (such as transistors and op amps) in addition to passive elements R, L and C. LC filters have been used in practical applications for more than eight decades.

Passive filters are widely used in power system for harmonic mitigation. In general, they have shunt branches consisting of passive elements; such as inductors and capacitors which are respectively tuned to the predominant harmonics. Design procedure for this type of filter is also very simple.

The design of a passive filter requires a precise knowledge of the harmonic producing load and of the power system. Because passive filters always provide reactive compensation to a degree dictated by the volt-ampere size and voltage of the capacitor bank used, they can in fact be designed for the double purpose of providing the filtering action and compensating power factor to the desired level.

Thus, passive filter design must take into account expected growth in harmonic current source or load reconfiguration because it can otherwise be exposed to overloading, which can rapidly develop into extreme overheating and thermal breakdown.

There are four types of filters whether passive or active:

- Low-pass filters
- High-pass filters
- Band-pass filters

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• Band-stop filters

A low-pass filter passes low frequencies and stops high frequencies. A high-pass filter passes high frequencies and rejects low frequencies. A band-pass filter passes frequencies within a frequency band and blocks or attenuates frequencies outside the band. A band-stop filter passes frequencies outside a frequency band and blocks or attenuates frequencies within the band

Low Pass Filter:

The prototype T and π low pass filter sections are as shown in the Fig. 9.3.

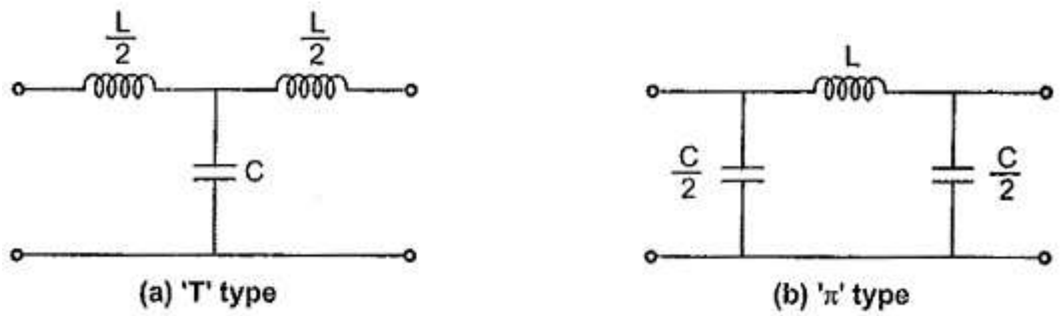

Fig. 9.3 Prototype T and π low pass filter sections

Design Impedance (R_0):

Here in low pass filter sections,

Total series arm impedance $Z_1 = j\omega L$

Total shunt arm impedance $Z_2 = -j/\omega C$

Hence, $Z_1 \cdot Z_2 = (j\omega L)(-j/\omega C) = L/C$ which is real and constant. Hence sections are constant K sections so we can write,

$$R_0^2 = Z_1 \cdot Z_2 = \frac{L}{C}$$

$$R_0 = \sqrt{\frac{L}{C}} \quad \ldots (1)$$

Reactance Curves and Cut-off Frequency Expression:

As both T and π sections have same cut-off frequency, it is sufficient to calculate f_c for the 'T' section only.

$$Z_1 = (j\omega L) \quad \therefore \quad X_1 = \omega L$$

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$$Z_2 = \frac{-j}{\omega C} \qquad \therefore \quad X_2 = \frac{-1}{\omega C}$$

$$\frac{X_1}{4} + X_2 = \frac{\omega L}{4} - \frac{1}{\omega C}$$

The reactance curves are as shown in the Fig. 9.4.

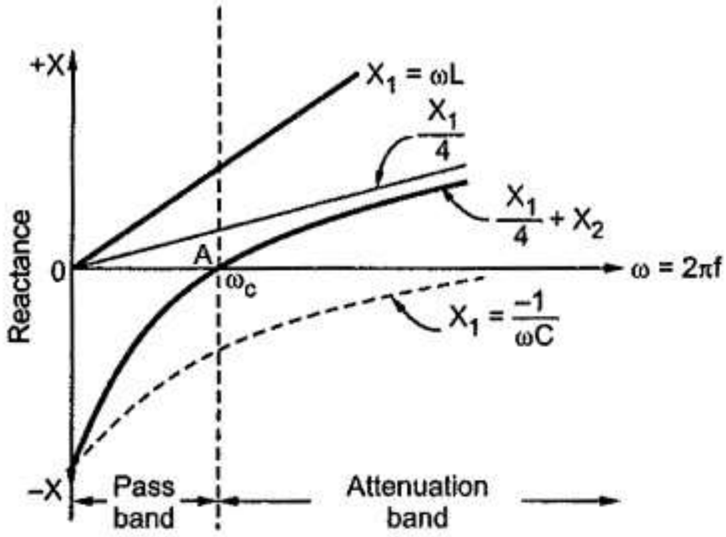

Fig. 9.4 Reactance-frequency sketch for prototype low pass filter

From above characteristic it is clear that all the reactance curves have positive slope as all curves slope upward to the right side with increasing ω.

The curves are on opposite sides of the frequency axis upto point A; while on the same side, from point A on wards. Hence all the frequencies upto point A give pass band and above point A give stop band. Thus point A marks cut-off frequency given by $\omega = \omega_c$.

At point A, $\omega = \omega_c$, the curve for $(X_1/4 + X_2)$ crosses the frequency axis, hence we can write,

$$\frac{\omega_c L}{4} - \frac{1}{\omega_c C} = 0$$

$$\frac{\omega_c L}{4} = \frac{1}{\omega_c C}$$

$$\omega_c^2 = \frac{4}{LC} \qquad \dots (2)$$

$$\omega_c = \frac{2}{\sqrt{LC}} \quad \text{or}$$

$$f_c = \frac{1}{\pi\sqrt{LC}} \qquad \dots (3)$$

The algebraic approach to calculate cut-off frequency is as follows.

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$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$= \sqrt{\frac{-\omega^2 L^2}{4} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}} \cdot \sqrt{1 - \frac{\omega^2 LC}{4}}$$

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}} \quad \ldots (4)$$

From above expression it is clear that, Z_{0T} is real if $\omega^2 LC/4 < 1$ and imaginary if $\omega^2 LC/4 > 1$. Hence condition $\omega^2 LC/4 - 1 = 0$ gives expression,

$$\omega^2 = \frac{4}{LC} \quad \text{or} \quad \omega = \frac{2}{\sqrt{LC}}$$

Thus, above prototype section passes all frequencies below $\omega = 2/\sqrt{LC}$ while attenuates all frequencies above this value. Therefore cut-off frequency of low pass filter is given by

$$\omega_c = \frac{2}{\sqrt{LC}} \quad \text{or} \quad f_c = \frac{1}{\pi\sqrt{LC}}$$

Above frequency comes out to be the same as calculated by reactance sketch method.

Variation of Z_{0T} and $Z_{0\pi}$ with Frequency:

Consider expression

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2 LC}{4}}$$

From equation (2), we can write

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega^2}{\omega_c^2}} \quad \text{or}$$

$$Z_{0T} = R_0 \sqrt{1 - \left(\frac{f}{f_c}\right)^2} \quad \ldots (5)$$

Similarly we can write,

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$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}}$$

$$Z_{0\pi} = \frac{R_0^2}{R_0\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

Hence

$$Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \quad \ldots (6)$$

From equation (5), it is clear that as frequency increases from 0 to f_c, Z_{0T} decreases from R_0 to 0 in passband. For π section, from equation (6), it is clear that in pass band as frequency increases for 0 to f_c, $Z_{0\pi}$ increases from R_0 to ∞.
The variation of Z_{0T} and $Z_{0\pi}$ with frequency is as shown in the Fig. 9.5.

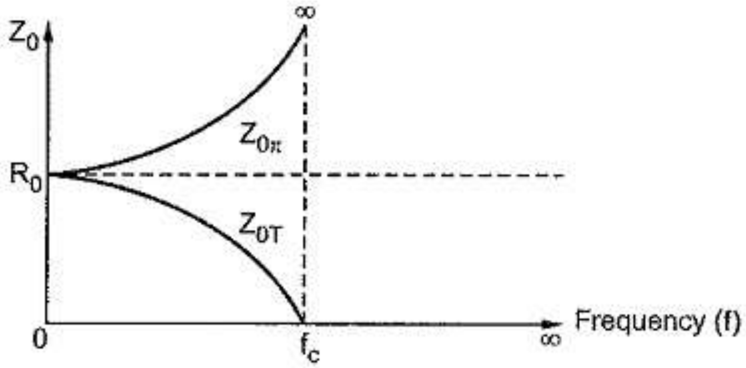

Fig. 9.5 Variation of characteristic impedance with frequency

Variation of Attenuation Constant α with Frequency:

In pass band attenuation is zero. In stop band attenuation is given by,

$$\alpha = 2\cosh^{-1}\left(\frac{f}{f_c}\right) \quad \ldots (7)$$

In stop band, as frequency f increases above f_c, attenuation also increases. The variation of α with frequency is as shown in the Fig. 9.6.

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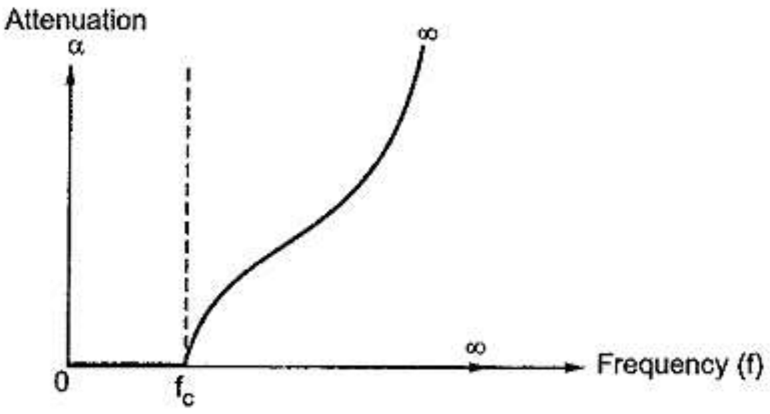

Fig. 9.6 Variation of attenuation constant with frequency

Variation of Phase Constant β with Frequency:

In stop band, phase constant β is always equal to π radian. In pass band where α = 0, the phase constant β is given by

$$\beta = 2 \sin^{-1}\left(\frac{f}{f_c}\right) \quad \ldots (8)$$

As frequency increases from 0 to f_c, β also increases from 0 to π radian. The variation of β with frequency is as shown in the Fig. 9.7.

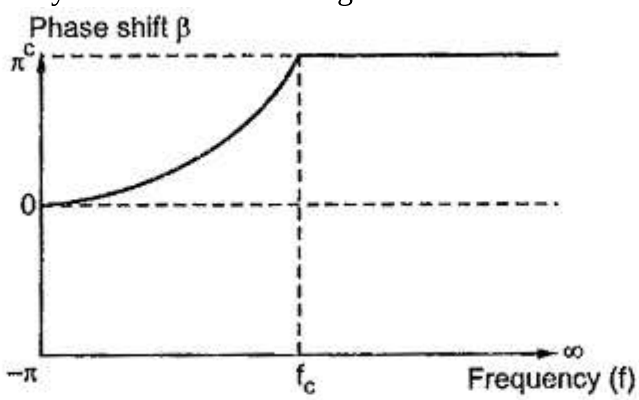

Fig. 9.7 Variation of phase constant with frequency

Design Equations of Prototype Low Pass Filter:

The design impedance R_0 and cut-off frequency f_c can be given in terms of L and C as follows.

$$R_0 = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

Dividing equation for R_0 by f_c, we get,

$$L = \frac{R_0}{(\pi f_c)} \quad \ldots (9)$$

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Multiplying equation for R_0 and f_c we get,

$$C = \frac{1}{(\pi f_c) R_0} \quad \ldots (10)$$

Equations (9) and (10) are called design equations for prototype low pass filter sections.

High Pass Filter:

The prototype high pass filter T and π sections are as shown in the Fig. 9.9.

Fig. 9.9 Prototype T and π high pass filter sections

Design Impedance (R_0):

Total series arm impedance $Z_1 = -j/\omega C$
Total shunt arm impedance $Z_2 = j\omega L$
Hence, $Z_1 \cdot Z_2 = (-j/\omega C)(j\omega L) = L/C$ which is real and constant. Hence above sections are constant K sections. So we can write,

$$R_0^2 = Z_1 \cdot Z_2 = \frac{L}{C}$$

$$R_0 = \sqrt{\frac{L}{C}} \quad \ldots (1)$$

$$Z_1 = \frac{-j}{\omega C} \qquad \therefore \quad X_1 = \frac{-1}{\omega C}$$

$$Z_2 = (j\omega L) \qquad \therefore \quad X_2 = \omega L$$

$$\frac{X_1}{4} + X_2 = \frac{-1}{4\omega C} + \omega L = \omega L - \frac{1}{4\omega C}$$

Reactance Curves and Expression for Cut-off Frequency:

As both T and π sections have same cut-off frequency, it is sufficient to calculate the cut-off frequency for the T section only.

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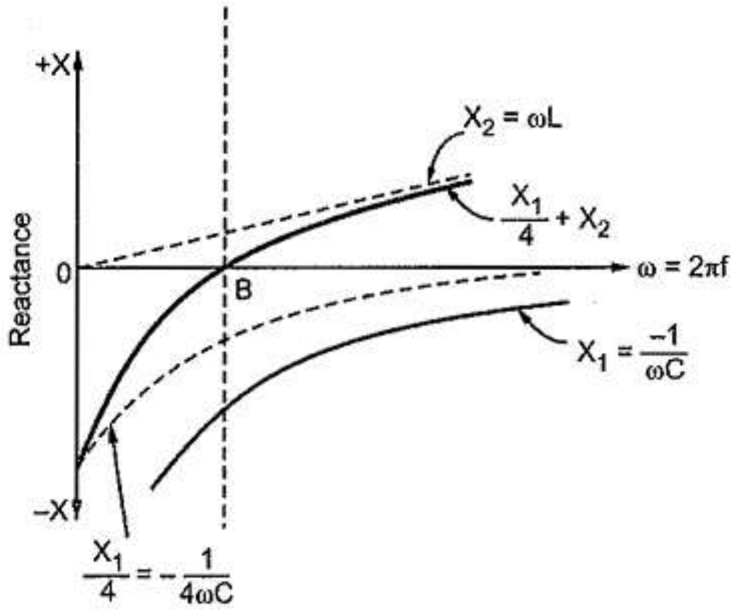

Fig. 9.10 Reactance-frequency sketch for prototype high pass filter

The reactance curves are as shown in the Fig. 9.10.

From above characteristics it is clear that all the reactance curves have positive slope as all curves slope upward to the right side with increasing ω.

Here the curves are on the same side of the horizontal axis up to the point B, giving a stop band. For frequencies above point B, the curves are on opposite sides of the axis, giving pass band. Thus, point B gives cut-off frequency, represented as $\omega = \omega_c$.
At point B, $\omega = \omega_c$, the curve for $(X_1/4 + X_2)$ crosses the frequency axis, hence we can write,

$$\omega_c L - \frac{1}{4\omega_c C} = 0$$

$$\omega_c L = \frac{1}{4\omega_c C}$$

$$\omega_c^2 = \frac{1}{4\,LC} \quad \ldots (2)$$

$$\omega_c = \frac{1}{2\sqrt{LC}} \quad \text{or}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}} \quad \ldots (3)$$

The algebraic approach to calculate cut-off frequency is as follows.

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$$Z_{0T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}} \times \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$Z_{0T} = R_0 \sqrt{1 - \frac{1}{4\omega^2 LC}} \quad \ldots (4)$$

From above expression it is clear that, Z_{0T} is real if $1/4\omega^2LC < 1$ and imaginary if $1/4\omega^2LC > 1$. Hence condition $1 - 1/4\omega^2LC = 0$ gives expression,

$$\omega^2 = \frac{1}{4LC} \text{ or } \omega = \frac{1}{2\sqrt{LC}}$$

Thus, above prototype section passes all frequencies above $\omega = 1/2\sqrt{LC}$ while attenuates all frequencies below this value. Therefore cut-off frequency of high pass filter is given by

$$\omega_c = \frac{1}{2\sqrt{LC}} \text{ or } f_c = \frac{1}{4\pi\sqrt{LC}}$$

Above frequency comes out to be same as frequency calculated by reactance sketch method.

Variation of Z_{0T} and $Z_{0\pi}$ with Frequency:

Consider expression for Z_{0T} as

$$Z_{0T} = R_0 \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

From equation (2) we can write,

$$Z_{0T} = R_0 \sqrt{1 - \frac{\omega_c^2}{\omega^2}} \quad \text{or}$$

$$Z_{0T} = R_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad \ldots (5)$$

Similarly we can write,

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{R_0^2}{R_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Hence,

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$$Z_{0\pi} = \frac{R_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad \ldots (6)$$

From equation (5), it is clear that as frequency f increases from f_c to ∞ in pass band, Z_{0T} also increases from 0 to R_0. For π section, from equation (6), it is clear that as frequency increases from f_c to ∞, Z_0 decreases from ∞ to R_0 in pass band. The variation of Z_{0T} and $Z_{0\pi}$ with frequency is as shown in Fig. 9.11.

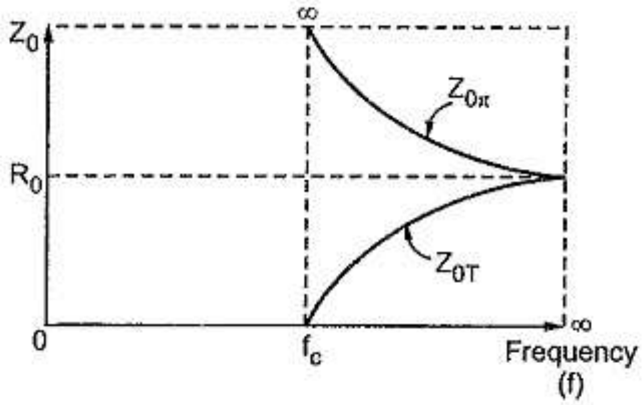

Fig. 9.11 Variation of characteristic impedance with frequency

Variation of Attenuation Constant (α) with Frequency:

In pass band, attenuation is zero ($\alpha = 0$). In stop band attenuation is given by

$$\alpha = 2\cosh^{-1}\left(\frac{f_c}{f}\right) \quad \ldots (7)$$

In stop band, as frequency f increases from 0 to f_c, attenuation decreases from ∞ to 0. The variation of attenuation constant α with frequency is as shown in the Fig. 9.12.

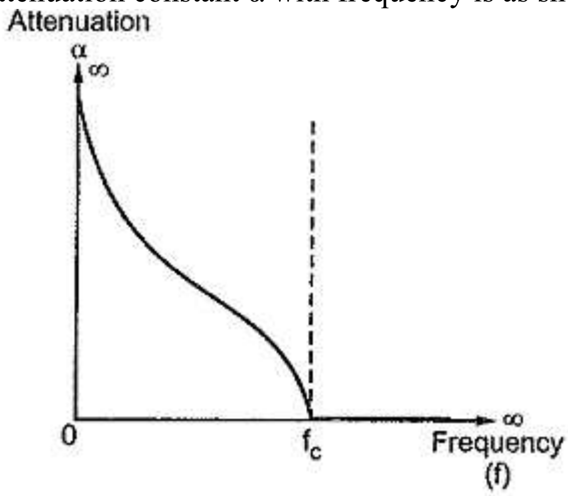

Fig. 9.12 Variation of attenuation constant (α) with frequency

Variation of Phase Constant β with Frequency:

In stop band, phase constant β is always π radian. In pass band where $\alpha = 0$, the phase angle β is given by

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$$\beta = 2\sin^{-1}\left(\frac{f_c}{f}\right) \qquad \ldots (8)$$

From the above equation it is clear that in Pass Band when frequency f increases from f_c to ∞, β decreases to 0. The variation of phase constant β with frequency is as shown in the Fig. 9.13.

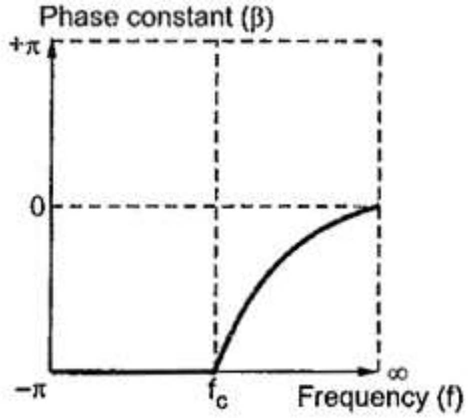

Fig. 9.13 Variation of phase constant with frequency

Design Equations of Prototype High Pass Filter:

The design impedance R_0 and cut-off frequency f_c for high pass filter section can be given in terms of L and C as follows

$$R_0 = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

Dividing equation for R_0 by f_c, we get,

$$L = \frac{R_0}{(4\pi f_c)} \qquad \ldots (9)$$

Multiplying equation for R_0 and f_c, we get,

$$C = \frac{1}{(4\pi f_c)R_0} \qquad \ldots (10)$$

Equation (9) and (10) are called design equations of prototype high pass filter sections.

Band Pass Filter:

Band pass filter pass a certain range of frequencies (called as **pass band**) while attenuate all other frequencies. Such band pass filters can be obtained by connecting low pass filter sections in cascade with high pass filter sections as shown in Fig. 9.15.
In above type of connection, the cut-off frequency of low pass filter section must be selected higher than that of high pass filter section.

Although cascade connection of low pass filter and high pass filter sections functions properly as band pass filter, it is more economical to combine both sections in one single filter section. An alternative form of band pass filter can be obtained either as a T or π section if series arm contains a series resonant circuit while the shunt arm contains a parallel resonant circuit as shown in the Fig. 9.16 (a) and (b).

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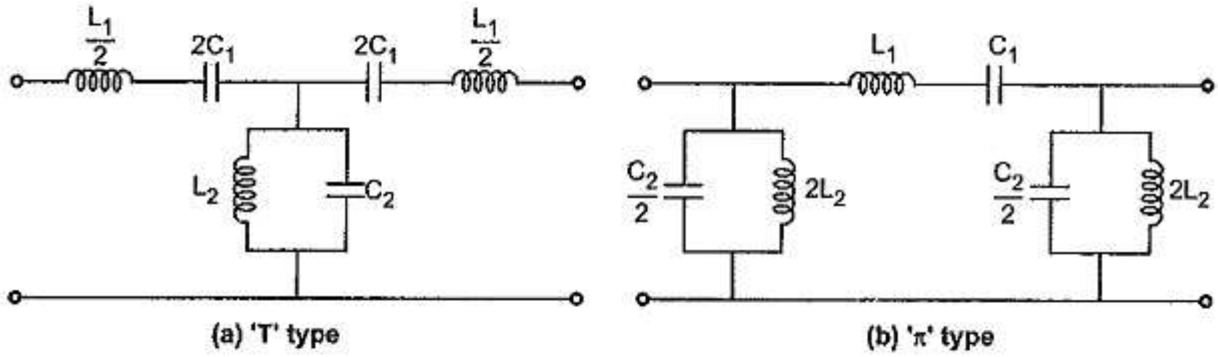

Fig. 9.16 Prototype T and π band pass filter sections

The band pass filter characteristics can be obtained by using conventional band pass filter (either T or π type) as shown in the Fig. 9.16, if the series resonant frequency of the series arm is selected same as anti resonant frequency of the shunt arm. Consider T type band pass filter section as shown in the Fig. 9.16 (a). Let the frequency of series and shunt arm be ω_0 rad/sec. Then, for series arm, frequency of resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{\left(\frac{L_1}{2}\right)(2C_1)}} = \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_0^2 L_1 C_1 = 1 \qquad \text{... (1)}$$

Similarly for shunt arm, frequency of anti resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{(L_2)(C_2)}}$$

$$\omega_0^2 L_2 C_2 = 1 \qquad \text{... (2)}$$

From equations (1) and (2), for same resonant frequencies of series and shunt arms we can write,

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \qquad \text{... (3)}$$

Design Impedance (R_0):

Total series arm impedance Z_1 is given by

$$Z_1 = j\omega L_1 + \left(\frac{-j}{\omega C_1}\right) = j\left[\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right] \qquad \text{... (4)}$$

Similarly, total shunt arm impedance Z_2 is given by

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$$Z_2 = (j\omega L_2) \,||\, \left(\frac{-j}{\omega C_2}\right) = \frac{(j\omega L_2)\left(\frac{-j}{\omega C_2}\right)}{j\omega L_2 - \frac{j}{\omega C_2}}$$

$$Z_2 = \frac{\frac{L_2}{C_2}}{\frac{j\left(\omega^2 L_2 C_2 - 1\right)}{\omega C_2}}$$

$$Z_2 = \frac{-j\omega L_2}{\left(\omega^2 L_2 C_2 - 1\right)} \qquad \text{... (5)}$$

Hence, $Z_1 Z_2 = L_2/C_1 = L_1/C_2$ which is real and constant. Hence above sections are constant k sections. So we can write,

$$R_0^2 = Z_1 Z_2 = \frac{L_2}{C_1} = \frac{L_1}{C_2}$$

$$R_0 = \sqrt{\frac{L_2}{C_1}} = \sqrt{\frac{L_1}{C_2}} \qquad \text{... (6)}$$

Reactance Curves and Expressions for Cut-off Frequencies:

To verify the band pass characteristics, let $Z_1 = j X_1$ and $Z_2 = j X_2$. Similar to the reactance curves drawn for low pass filter section and high pass filter section, sketching reactances X_1 and $(X_1/4 + X_2)$ against frequency f as shown in the Fig. 9.17.

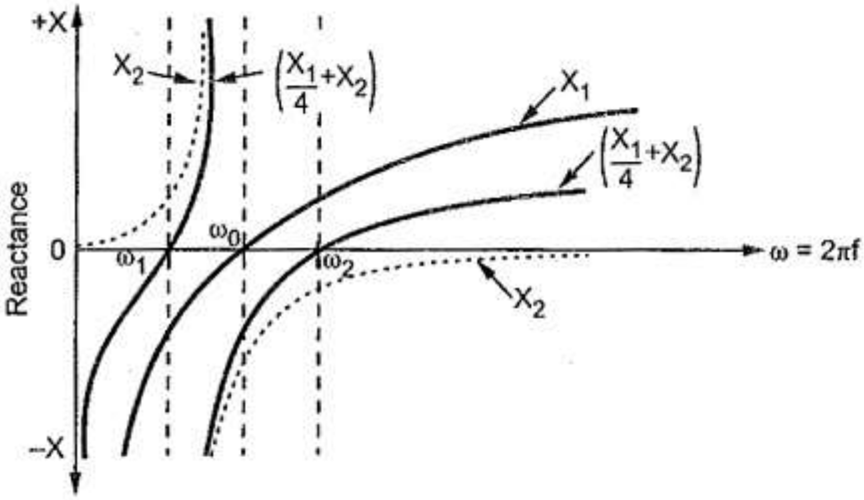

Fig. 9.17 Reactance frequency sketch for constant k band pass filter

From the above characteristics it is clear that the reactance curves for X_1 and $(X_1/4 + X_2)$ are on the same sides the axis below f_1 and above f_2. At the same time, the reactance curves between f_1 and f_2 are on opposite sides of frequency axis. Thus frequencies between f_1 and f_2 constitute a pass band ; while the frequencies below f_1 and above f_2 give stop band. Hence the section considered shows band pass filter characteristics where f_1 and f_2 are lower and upper cut-off frequencies of the filter.

In band pass filter, condition for cut-off frequency is,

$$\frac{Z_1}{4} + Z_2 = 0$$

$$Z_1 = -4\ Z_2$$

$$Z_1^2 = -4\ Z_1\ Z_2$$

But from the condition of constant-k filter section, $Z_1\ Z_2 = R^2_0$

$$Z_1^2 = -\ 4\ R_0^2$$

$$Z_1 = \pm\ j\ (2\ R_0) \qquad \text{... (7)}$$

From above equation (7) it is clear that the value of the series arm impedance Z_1 can be obtained at two different cut-off frequencies namely f_1 and f_2. So at $f = f_1$, $Z_1 = -\ j(2\ R_0)$ and at $f = f_2$, $Z_1 = +\ j(2\ R_0)$. Thus impedance Z_1 at f_1, i.e. lower cut-off frequency, is negative of the impedance Z_1 at f_2 i.e. upper cut-off frequency. Hence we can write,

$$\omega_2\ L_1 - \frac{1}{\omega_2\ C_1} = -\left(\omega_1\ L_1 - \frac{1}{\omega_1\ C_1}\right)$$

$$\frac{\omega_2^2\ L_1\ C_1 - 1}{\omega_2\ C_1} = -\left(\frac{\omega_1^2\ L_1\ C_1 - 1}{\omega_1\ C_1}\right)$$

$$\omega_2^2\ L_1\ C_1 - 1 = \frac{\omega_2}{\omega_1}\left(1 - \omega_1^2\ L_1\ C_1\right) \qquad \text{... (8)}$$

But from equation (1) we can write,

$$\omega_0^2 = \frac{1}{L_1\ C_1} \quad \text{or} \quad L_1\ C_1 = \frac{1}{\omega_0^2}$$

Substituting value of ($L_1\ C_1$) in above equation (8), we can write,

$$\frac{\omega_2^2}{\omega_0^2} - 1 = \frac{\omega_2}{\omega_1}\left(1 - \frac{\omega_1^2}{\omega_0^2}\right)$$

$$\left(1 - \frac{\omega_1^2}{\omega_0^2}\right) = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2}{\omega_0^2} - 1\right)$$

Simplifying above equation,

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$$\frac{\omega_0^2 - \omega_1^2}{\omega_0^2} = \frac{\omega_1}{\omega_2}\left(\frac{\omega_2^2 - \omega_0^2}{\omega_0^2}\right)$$

$$\omega_2 \omega_0^2 - \omega_2 \omega_1^2 = \omega_1 \omega_2^2 - \omega_1 \omega_0^2$$

$$\omega_2 \omega_0^2 + \omega_1 \omega_0^2 = \omega_1 \omega_2^2 + \omega_2 \omega_1^2$$

$$\omega_0^2 (\omega_2 + \omega_1) = \omega_1 \omega_2 (\omega_2 + \omega_1)$$

$$\omega_0^2 = \omega_1 \omega_2$$

$$f_0^2 = f_1 \cdot f_2$$

$$f_0 = \sqrt{f_1 \cdot f_2} \quad \text{... (9)}$$

Hence, above equation (9) indicates that frequency of resonance of the individual arms is the geometric mean of two cut-off frequencies:

Variation of Z_{0T} and $Z_{0\pi}$, Attenuation Constant (α) and Phase Constant (β) with Frequency:

The variations of Z_{0T} and $Z_{0\pi}$, attenuation constant (α) and phase shift (β) with frequency are as shown in the Fig. 9.18. (a), (b) and (c). Consider that the design impedance of band pass filter is R_0 and cut-off Frequencies are f_1 and f_2.

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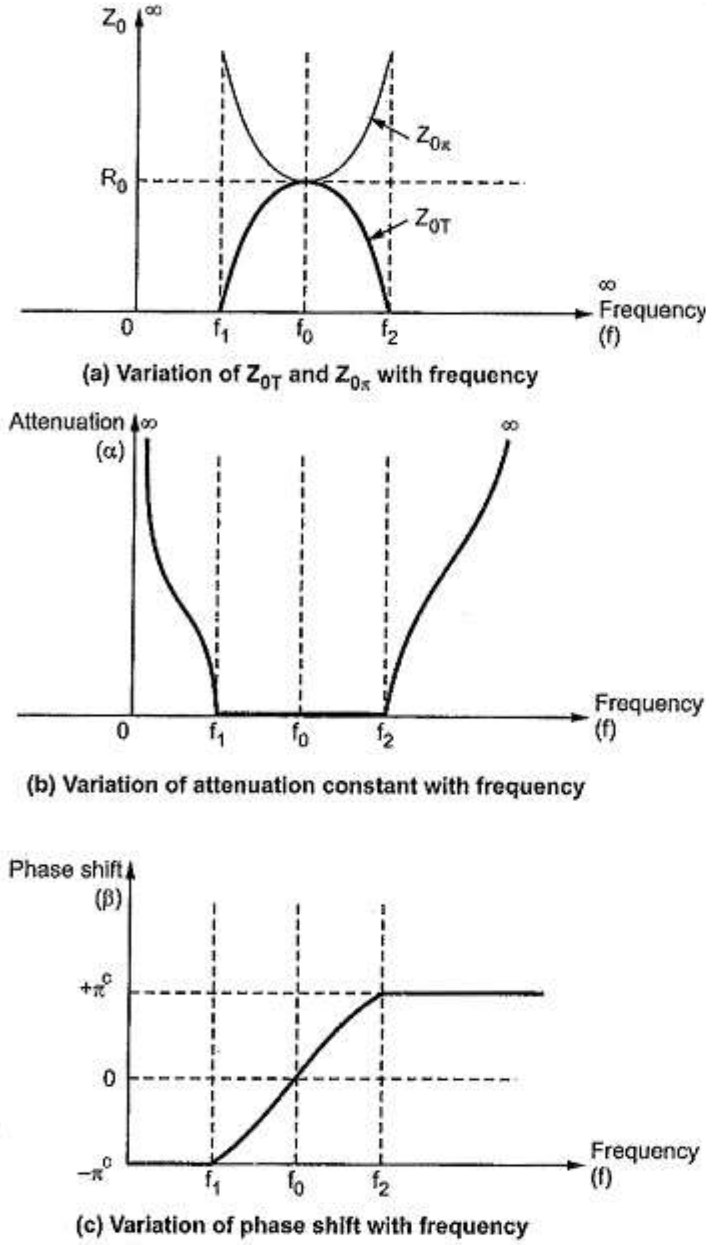

(a) Variation of Z_{0T} and $Z_{0\pi}$ with frequency

(b) Variation of attenuation constant with frequency

(c) Variation of phase shift with frequency

Fig. 9.18

Design Equations:

Consider that the filter is terminated in design impedance R_0 and the cut-off frequencies are f_1 and f_2.

Then from equation (7), at the lower cut-off frequency f_1, we can write,

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$$j\omega_1 L_1 - \frac{j}{\omega_1 C_1} = -j(2R_0)$$

$$\frac{1}{\omega_1 C_1} - \omega_1 L_1 = 2R_0$$

$$1 - \omega_1^2 L_1 C_1 = 2R_0(\omega_1 C_1)$$

$$1 - \frac{\omega_1^2}{\omega_0^2} = (2\omega_1 C_1) R_0 \; .... \because \omega_0^2 = \frac{1}{L_1 C_1}$$

$$1 - \frac{(2\pi f_1)^2}{(2\pi f_0)^2} = 2(2\pi f_1) C_1 R_0$$

$$1 - \frac{f_1^2}{f_0^2} = 4\pi R_0 f_1 C_1$$

$$f_0^2 = f_1 f_2$$

$$1 - \frac{f_1^2}{f_1 f_2} = 4\pi R_0 f_1 C_1$$

$$\frac{f_2 - f_1}{f_2} = 4\pi R_0 f_1 C_1$$

$$C_1 = \frac{(f_2 - f_1)}{4\pi R_0 (f_1 f_2)} \qquad ... (10)$$

But for band pass filter constant k section

$$f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

$$f_0^2 = \frac{1}{4\pi^2 L_1 C_1}$$

$$L_1 = \frac{1}{4\pi^2 C_1 f_0^2}$$

Substituting the value of C_1 from equation (10),

$$L_1 = \frac{1}{4\pi^2 f_0^2 \left[\frac{(f_2 - f_1)}{4\pi R_0 f_2 f_1}\right]}$$

As $f^2_0 = f_1 f_2$, we get

$$L_1 = \frac{R_0}{\pi(f_2 - f_1)} \qquad ... (11)$$

From equation (6), we can write,

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$$R_0^2 = \frac{L_1}{C_2}$$

$$C_2 = \frac{L_1}{R_0^2}$$

Substituting value of L_1 from equation (11),

$$C_2 = \frac{\frac{R_0}{\pi(f_2 - f_1)}}{R_0^2}$$

$$C_2 = \frac{1}{\pi R_0 \, (f_2 - f_1)} \quad \text{... (12)}$$

From equation.(6), we can write,

$$R_0^2 = \frac{L_2}{C_1}$$

$$L_2 = C_1 \cdot R_0^2$$

Substituting value of C_1 from equation (10),

$$L_2 = \frac{(f_2 - f_1)}{4\pi R_0 (f_1 \; f_2)} R_0^2$$

$$L_2 = \frac{R_0 \, (f_2 - f_1)}{4\pi f_1 \; f_2} \quad \text{... (13)}$$

Equations (10) to (13) are called design equations of prototype band pass filter sections.

Band Stop Filter:

Band Stop Filter stop a range of frequencies between two cut-off frequencies f_1 and f_2 while pass all the frequencies below f_1 and above f_2. Thus range of frequencies between f_1 and f_2 constitutes a stop band in which attenuation to the frequencies is infinite ideally. The frequencies below f_1 and above f_2 constitute two separate pass bands in which attenuation to the frequencies is zero ideally.

The Band Stop Filter can be obtained by connecting low pass filter and high pass filter sections in parallel where cut-off frequency of the low pass filter section is less than that of the high pass filter section. But the economical form of the band elimination filter can be obtained by combining the low pass and high pass filter section if series, arm contains parallel resonant circuit while shunt arm contains series resonant circuit as shown in the Fig. 9.20 (a) and (b).

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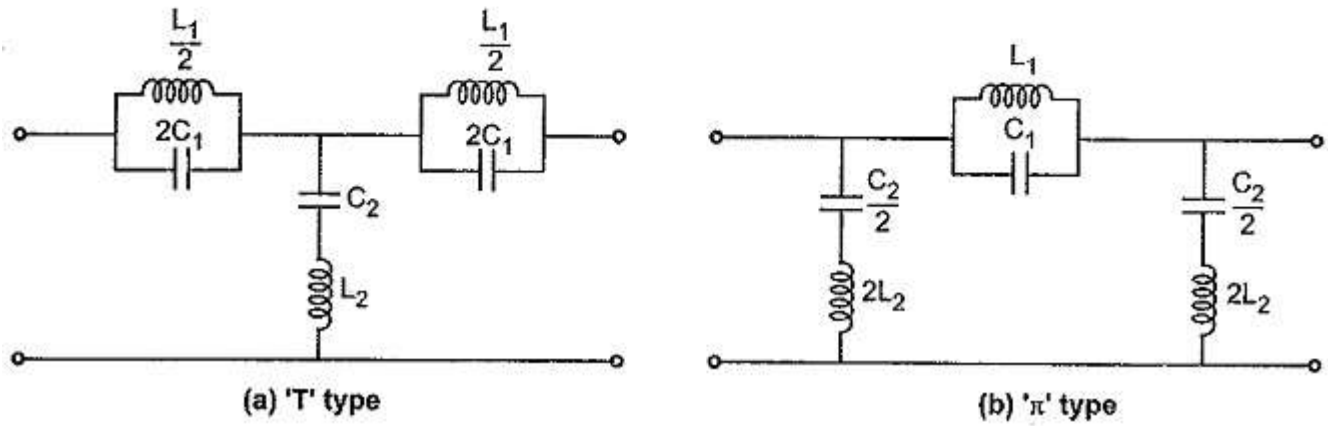

Fig. 9.20 Prototype T and π band elimination filter sections

The band elimination characteristics can be obtained by using conventional Band Stop Filter (either T or π type) as shown in the Fig. 9.20, if the series resonant frequency of the shunt arm is selected same as the parallel resonant frequency of the series arm. Consider 'T' type band elimination filter section as shown in the Fig. 9.20(a).

Let the frequency of the series and shunt arm be ω_0 rad/sec. Then for series arm, frequency of anti-resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{\left(\frac{L_1}{2}\right)(2C_1)}} = \frac{1}{\sqrt{L_1 C_1}}$$

$$\omega_0^2 L_1 C_1 = 1 \quad \text{... (1)}$$

Similarly, for shunt arm, frequency of resonance is given by,

$$\omega_0 = \frac{1}{\sqrt{(L_2)(C_2)}}$$

$$\omega_0^2 L_2 C_2 = 1 \quad \text{... (2)}$$

From equations (1) and (2), for same resonant frequencies of series and shunt arm resonant circuit we can write,

$$\omega_0^2 L_1 C_1 = 1 = \omega_0^2 L_2 C_2$$

$$L_1 C_1 = L_2 C_2 \quad \text{... (3)}$$

Design Impedance (R_0):

Total series arm impedance is given by,

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$$Z_1 = \frac{(j\omega L_1)\left(-\frac{j}{\omega C_1}\right)}{j\omega L_1 - \frac{j}{\omega C_1}} = \frac{\omega L_1}{j\left(\omega^2 L_1 C_1 - 1\right)}$$

$$Z_1 = \frac{-j\omega L_1}{\left(\omega^2 L_1 C_1 - 1\right)}$$

$$Z_1 = \frac{\omega L_1}{\left(1-\omega^2 L_1 C_1\right)} \quad \ldots (4)$$

Similarly, total shunt arm impedance Z_2 is given by,

$$Z_2 = j\omega L_2 - \frac{j}{\omega C_2} = j\left(\frac{\omega^2 L_2 C_2 - 1}{\omega C_2}\right) \quad \ldots (5)$$

Hence $Z_1 Z_2 = L_2/C_1 = L_2/C_2$ which is real and constant. Hence above sections are constant k sections. So we can write,

$$R_0^2 = Z_1 Z_2 = \frac{L_1}{C_2} = \frac{L_2}{C_1}$$

$$R_0 = \sqrt{\frac{L_1}{C_2}} = \sqrt{\frac{L_2}{C_1}} \quad \ldots (6)$$

Reactance Curves and Expressions for Cut-off Frequencies:

To verify the band elimination characteristics, let $Z_1 = j X_1$ and $Z_2 = j X_2$. The reactance curves of X_1 and $X_1/4 + X_2$ against frequency are as shown in the Fig. 9.21.

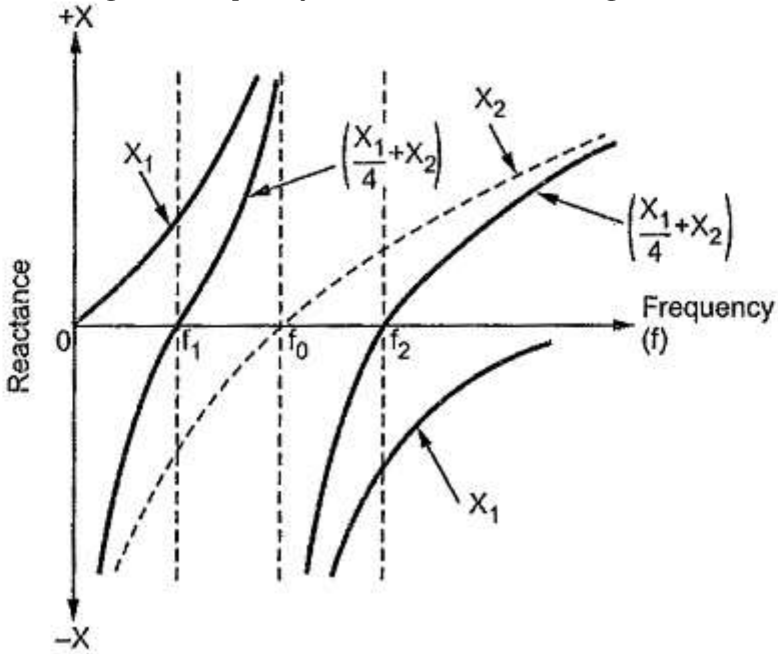

Fig. 9.21 Reactance frequency sketch for constant K band elimination filter

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From the above characteristics it is clear that the reactance curves for X_1 and $(X_1/4 + X_2)$ are on the same sides of the frequency axis between f_1 and f_2 which indicates stop band. These curves are on opposite sides of the axis below f_1 and above f_2 which indicates two pass band. Hence for the given section, the characteristics are of band elimination filter where f_1 and f_2 are the cut-off frequencies.
In Band Stop Filter, the condition for cut-off frequencies is given by

$$\frac{Z_1}{4} + Z_2 = 0$$

$$\frac{Z_1}{4} = -Z_2$$

$$Z_1^2 = -4\ Z_1 Z_2$$

But from the condition of constant K filter section, $Z_1Z_2 = R^2_0$

$$Z_1^2 = -4 R_0^2$$

$$Z_1 = \pm j\,(2\ R_0) \qquad \text{... (7)}$$

From above equation it is clear that the value of the series arm impedance Z_1 can be obtained at two different cut-off frequencies namely f_1 and f_2. So at $f = f_1$, $Z_1 = +\,j(2\ R_0)$ and at $f = f_2$, $Z_1 = -j(2\ R_0)$. Thus impedance Z_1 at f_1, i.e. at lower cut-off frequency, is negative of the impedance Z_1 at f_2 i.e. upper cut-off frequency. Hence we can write,

$$\frac{\omega_1 L_1}{1-\omega_1^2 L_1 C_1} = -\frac{\omega_2 L_1}{1-\omega_2^2 L_1 C_1}$$

$$1-\omega_2^2 L_1 C_1 = \frac{-\omega_2}{\omega_1}\left(1-\omega_1^2 L_1 C_1\right)$$

$$1-\omega_2^2 L_1 C_1 = \frac{\omega_2}{\omega_1}\left(\omega_1^2 L_1 C_1 - 1\right) \qquad \text{... (8)}$$

But from equation (1), frequency of resonance is given by

$$\omega_0^2 = \frac{1}{L_1 C_1} \quad \text{or} \quad L_1 C_1 = \frac{1}{\omega_0^2}$$

Substituting value of $(L_1 C_1)$ in above equation, we can write,

$$1-\frac{\omega_2^2}{\omega_0^2} = \frac{\omega_2}{\omega_1}\left(\frac{\omega_1^2}{\omega_0^2}-1\right)$$

Simplifying above equation,

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$$\frac{\omega_0^2 - \omega_2^2}{\omega_0^2} = \frac{\omega_2}{\omega_1}\left(\frac{\omega_1^2 - \omega_0^2}{\omega_0^2}\right)$$

$$\omega_1 \omega_0^2 - \omega_1 \omega_2^2 = \omega_2 \omega_1^2 - \omega_2 \omega_0^2$$

$$\omega_1 \omega_0^2 + \omega_2 \omega_0^2 = \omega_2 \omega_1^2 + \omega_1 \omega_2^2$$

$$\omega_0^2 (\omega_1 + \omega_2) = \omega_1 \omega_2 (\omega_2 + \omega_1)$$

$$\omega_0^2 = \omega_1 \omega_2$$

$$f_0^2 = f_1 f_2$$

$$f_0 = \sqrt{f_1 f_2} \quad \text{... (9)}$$

Hence, above equation (9) indicates that in band elimination filter, the frequency of resonance of the individual arms is the geometric mean of two cut-off frequencies.

Variation of Z_{0T} and $Z_{0\pi}$, Attenuation Constant (α), Phase Constant (β) with Frequency:

The variations of Z_{0T} and $Z_{0\pi}$,attenuation constant (α) and phase shift (β) are as shown in the Fig. 9.22 (a), (b) and (c) respectively. Consider that f_1 and f_2 are two cut-off frequencies and R_0 is the design impedance of the Band Stop Filter.

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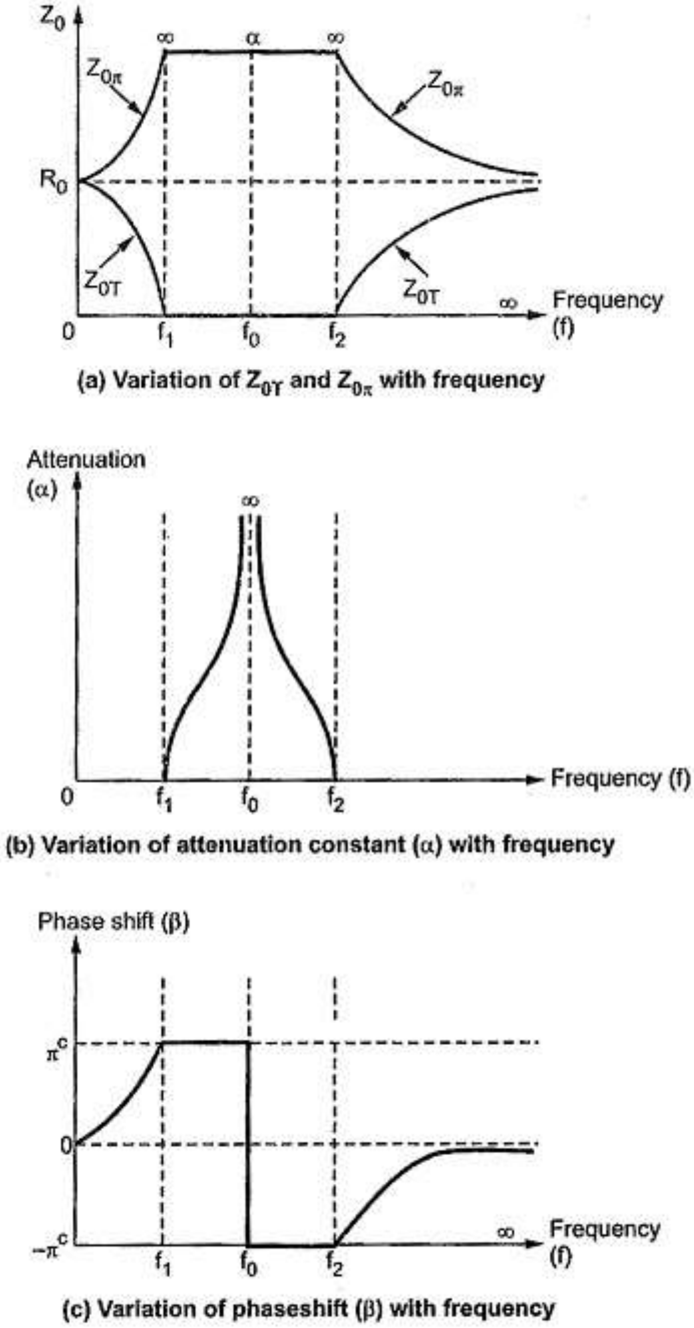

Fig. 9.22

Design Equations:

Consider that a band elimination filter with two cut-off frequencies f_1 and f_2 is terminated in design impedance R_0. Then, from equation (7), at lower cut-off frequency f_1, we can write,

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$$j\left(\frac{\omega_1 L_1}{1-\omega_1^2 L_1 C_1}\right) = +j(2R_0)$$

$$\omega_1 L_1 = 2R_0\left(1-\omega_1^2 L_1 C_1\right)$$

$$\omega_1 L_1 = 2R_0\left(1-\frac{\omega_1^2}{\omega_0^2}\right) \text{.........} \because \quad L_1 C_1 = \frac{1}{\omega_0^2}$$

$$\omega_1 L_1 = 2R_0\left(1-\frac{\omega_1^2}{\omega_1 \omega_2}\right) \text{..........} \omega_0^2 = \omega_1 \omega_2$$

$$L_1 = 2R_0\left(\frac{\omega_2-\omega_1}{\omega_1 \omega_2}\right)$$

$$L_1 = \frac{2R_0(2\pi)(f_2-f_1)}{4\pi^2(f_1 f_2)}$$

$$L_1 = \frac{R_0(f_2-f_1)}{\pi f_1 f_2} \qquad \text{... (10)}$$

For band elimination filter constant K section, frequency of resonance in series arms is given by,

$$f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

$$f_0^2 = \frac{1}{4\pi^2 L_1 C_1}$$

$$C_1 = \frac{1}{4\pi^2 f_0^2 L_1} = \frac{1}{4\pi^2(f_1 f_2) L_1} \text{.........} \because \quad f_0^2 = f_1 f_2$$

Substituting value of L_1 in above equation,

$$C_1 = \frac{1}{4\pi^2(f_1 f_2)\left[\frac{R_0(f_2-f_1)}{\pi(f_1 f_2)}\right]}$$

$$C_1 = \frac{1}{4\pi R_0 (f_2-f_1)} \qquad \text{... (11)}$$

From equation (6) we can write,

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$$R_0^2 = \frac{L_1}{C_2}$$

$$C_2 = \frac{L_1}{R_0^2}$$

Substituting value of L_1 from equation (10),

$$C_2 = \frac{1}{R_0^2}\left[\frac{R_0(f_2 - f_1)}{\pi(f_1 f_2)}\right]$$

$$C_2 = \frac{(f_2 - f_1)}{\pi R_0 (f_1 f_2)} \quad \text{... (12)}$$

Similarly from equation (6) we can write,

$$R_0^2 = \frac{L_2}{C_1}$$

$$L_2 = R_0^2 C_1$$

Substituting value of C_1 from equation (11),

$$L_2 = R_0^2\left[\frac{1}{4\pi R_0 (f_2 - f_1)}\right]$$

$$L_2 = \frac{R_0}{4\pi(f_2 - f_1)} \quad \text{... (13)}$$

Equations (10) to (13) are called design equations of prototype band elimination filter sections.

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